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A. Decimals

(A1) Place Value

Icebreaker:

If you currently have a bank account—what's the most money you've ever had in it?

(KEEP YOUR ANSWER TO YOURSELF! AND DON'T WRITE IT DOWN!)

When Julie is asked what's the most she's ever had in her current bank account, she just says,

“Four.”

Four *what*, though? 4 thousand? 4 hundred? 4 dimes?

When we write a number in decimal form, the value (or “**place value**”) of each digit is given by its position, either left or right of the decimal point.

Julie claims the most she's ever had in her current bank account is (*in dollars*):

billions	hundred millions	ten millions	millions	hundred thousands	ten thousands	4 thousands	,	0 hundreds	0 tens	0 units	.	1 tenths	0 hundredths
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How many billions does Julie have? Zero. How many millions? Zero. To rub it in, we could write zero in each place value to the left of the 4 in 4,000:

0 billions	0 hundred millions	0 ten millions	0 millions	0 hundred thousands	0 ten thousands	4 thousands	,	0 hundreds	0 tens	0 units	.	1 tenths	0 hundredths
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It's unusual to write extra zeros to the *left* of the largest non-zero place, on the left-hand side of the decimal point.

It's *very* common, however, to add zeros on the *right*-hand side of the decimal point.

How many *thousandths* does Julie have? How many *ten thousandths*?

Exercise 1. Write the number "*four hundred thousand thirty and one hundredth*":

millions	hundred thousands	ten thousands	thousands	hundreds	tens	units	.	tenths	hundredths	thousandths	ten thousandths	hundred thousandths	millionths
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A **decimal** is a number¹ written with a decimal point.

- A decimal may represent either an exact quantity or an approximation.



A decimal is **terminating** if there is some place value to the right of which all digits are 0.

- For example, “*thirty*” = 30.000... is terminating.
- We usually just write “30” and not “30.000...”

A **whole number** is a non-negative (≥ 0) number with all zeros to the right of the decimal point.

- We often don’t write the decimal point (or the zeros to the right of the decimal point) when we write a whole number.
- For example, the price of a \$409,450 house—the median price of a single-family house in Atlanta in 2021,² would typically not be written as \$409,450.00.

Exercise 2. Write the decimal form for each part in the space provided, then answer the questions.

- Is the decimal form of “*one third*” terminating?

- Is the decimal form of “*three halves*” terminating?

Note to international students:

- Online assignments will mark as incorrect “4.000” for “*four thousand*,” or “3,5” for “*three and five tenths*.”
- When writing any number in this class, please use commas (,) and periods (.) in the English style. Your instructor can answer any questions you may have about the “English style” of writing numbers.
- Also note that, in English usage, a “million” means 1,000,000, and a “billion” means a thousand millions = 1,000,000,000.

¹When we say “a decimal,” we mean the entire number—not just the part after the decimal point. For example, the decimal shown at the top of this page is 72.3 (not just .3).

²Source: [Roofstock.com](http://roofstock.com), a California company that sells rental houses in Atlanta to landlords “investors” seeking “strong, stable cash flows even through economic cycles” (Quote from front page of website, <http://roofstock.com/>). The website cites a 2020 [Atlanta Business Journal](#) article that says, “[A] major reason people rent, especially [M]illennials and Generation Z, is that they cannot afford to purchase a home in the area of town they want to live. Atlanta, like most major cities, is struggling to provide affordable multiunit housing solutions due to zoning and density restrictions in popular areas. Within the housing industry, this concept is known as the “missing middle.”” The “stability” of cash flows from Atlanta renters is presumably guaranteed by the fact of the ongoing [eviction crisis](#) (Source: AJC).

(A2) Rounding

To round to the nearest _____, follow these steps:
(INSERT PLACE VALUE HERE)

- Look at the digit one place to the right.

For example, if asked to round to the nearest hundred, look at the tens place.

- If 4 or less, round down.

For example:

137.50 rounded to the nearest hundred is _____

- If 5 or larger, round up.

For example:

137.50 rounded to the nearest unit is _____

Exercise 3. Round the number to each of the given place values.

	hundreds	tens	whole number	tenths	hundredths
(a) 65,743.18					
(b) 19.735					
(c) 8,456.1893					
(d) 318.2761904					

(A3) Arithmetic with Decimals

In this class, when a calculation involves decimals, we will almost always use a calculator.

Exercise 4. Verify that you know how to enter “*negative three*” on the calculator.

- (*Optional:*) Write a sentence in plain English explaining how to enter “*negative three*” on the calculator.

Exercise 5.

- Add “fifteen and negative two” on the calculator. *What is the largest place value in the result?*
- When we add two numbers, does the order matter?
That is, is it true that “x plus y” and “y plus x” are always equal, no matter what numbers x and y represent?

Exercise 6.

- Write “negative sixteen divided by negative three” in arithmetic symbols.
- Before calculating the result, predict whether the result will be SMALLER than 16 or LARGER than 16.
- Divide negative sixteen by negative three on the calculator, and write your result:
- Was your prediction correct?

Exercise 7.

- | | |
|-----------------------------|--------------------------------|
| (a) $0.32(2051) =$ | (f) $0.00873 \times 10^{-2} =$ |
| (b) $4.5 - 3.87 =$ | (g) $1.8(45) + 32 =$ |
| (c) $7.20 \div 24 =$ | (h) $(23 - 32) \div 1.8 =$ |
| (d) $17.93 - 18.2 =$ | (i) $1.07 \times 48.50 =$ |
| (e) $0.00873 \times 10^4 =$ | |

B. Decimals and Fractions.

(B1) Converting a Decimal to a Fraction

Ten dimes equals one dollar.



=



That is, a dime is *one tenth* of a dollar.

We can write “*one tenth*” as a decimal or as a fraction:

$$0.1 = \frac{1}{10} \quad (*)$$

Notice that neither side of the above equation (*) looks like money.

- After today, money must always be written in the American style in this class:

\$0.10

We read this as “*ten cents*.”

What is the number 0.01 as a fraction?

$$0.01 = \frac{1}{100}$$

What is the number 0.001 as a fraction?

$$0.001 = \frac{1}{\boxed{}}$$

To convert a terminating decimal to a fraction:

- Count the number of places—call it n —to the right of the decimal point.
- The denominator is $10^n =$ (“1” followed by n zeros).

For example, the denominator for 3.14 is $10^2 = 100$.

- The numerator is the decimal without a decimal point.

For example, $3.14 = \frac{314}{100}$.

(B2) Converting a Fraction to a Decimal

To convert a fraction to a decimal, all we need to know is that *the bar in a fraction means "divided by"*.

For example, the fraction $\frac{30}{20}$ means _____ divided by _____.

In symbols:

$$\frac{30}{20} = \boxed{} \div \boxed{}$$

To find the decimal form of $\frac{30}{20}$, just enter the above division into the calculator.

Exercise 8. Complete the table below by converting the given value to the other form. Round decimals to the nearest thousandth, if necessary.

	fraction	decimal
(a)	$\frac{5}{8}$	
(b)		0.035
(c)		0.400
(d)	$\frac{2}{3}$	
(e)		1.200
(f)	$\frac{9}{4}$	
(g)	$\frac{3}{7}$	

C. Fractions.

(C1) Numerator and Denominator

A **fraction** is an expression of the form $\frac{N}{D}$. It simply means, “ $N \div D$.”

- The “top” of the fraction, N , is called the **numerator**.
- The “bottom,” D , is called the **denominator**.

Coins that are worth different fractions of a dollar are said to have different **denominations**.

- For example, a dime is one tenth of a dollar, and a penny (a “cent”) is one hundredth of a dollar.
- Three guesses—what fraction of a dollar is a 25-cent coin worth?

The *numerator* of a fraction with a denominator of **10**—like

$$\frac{3}{10}$$

—*counts* the number of TENTHS—say, of a dollar (“*three dimes*”).

The *numerator* in

$$\frac{7}{100}$$

counts HUNDREDTHS—say, of a dollar (“*seven cents*”).

Exercise 9.

- Write “*five cents*” as a fraction of a dollar, using a denominator of 100.

- Can you write the same quantity using a different denominator?

- Old Spanish silver dollars were sometimes divided—literally, cut up—into eighths:



Can you write the amount “twenty-five (American) cents” as a fraction of a dollar, using a denominator of 8?



$$\frac{25}{100} =$$

(C2) Reducing a Fraction

To **reduce a fraction** means to rewrite it so that the denominator (*bottom*) is as small a natural number (1, 2, 3, ...) as possible, while the numerator (*top*) is either a whole number (0, 1, 2, 3, ...) or the negative of a whole number.

- The usual process for reducing a fraction is to find a number that evenly divides top and bottom.
- Dividing both the top and bottom by the same number is called **canceling**.
- Note that a fraction with a zero upstairs is allowed $\left(\frac{0}{k}\right)$, but a denominator of 0 is forbidden $\left(\frac{N}{0}\right)$.

Exercise 10.

- Reduce the fraction $\frac{75}{100}$.
- Reduce the fraction $\frac{85}{100}$.
- (*Optional:*) Write a sentence in plain English explaining what you did in the previous two parts.

Exercise 11. A **shilling** was a unit of money in the British Empire prior to 1971.

One shilling was equal to 12 pennies. . .



. . . just as one American dollar today is worth ten dimes.

- Write “four pennies” as a fraction of a shilling. Give your answer in reduced form.

Exercise 12. In Karl Marx’s *Capital*, Vol. 1, the standard wage for a worker is assumed to be 3 shillings for each 12-hour shift.

- Complete the table:

WAGES PAID:	3 shillings		18 shillings
HOURS WORKED:	12 hours	24 hours	

- Complete the equation:

$$\frac{3}{12} = \frac{\boxed{}}{24} = \frac{18}{\boxed{}}$$

- (Optional:) Explain in plain English what you did to find the missing numbers above.

- TRUE/FALSE: The reduced form of all three fractions is $\frac{1}{4}$.

D. Equal in Proportion.

Exercise 13. What is your hourly wage if you are paid as follows?

WAGES PAID:	\$60	\$90	\$120
HOURS WORKED:	4 hours	6 hours	8 hours

The amount you get paid (\$) and the amount you work (hr) are **equal in proportion**.

This means, if you get paid x dollars for y hours of work, and you get paid m dollars for n hours of work, then

$$\frac{x \text{ dollars}}{y \text{ hours}} = \frac{m \text{ dollars}}{n \text{ hours}}.$$

In the above example, we have

$$\frac{60}{4} = \frac{90}{6} = \frac{120}{8}.$$

Indeed, all these fractions reduce to $\frac{15}{1}$, which represents \$15 for 1 hour.

Another way of saying this is, “the RATIO of dollars to hours is 15 to 1.”

- We'll call an equation of the form

$$\frac{x}{y} = \frac{m}{n}$$

a **proportion equation**.

- The reduced form of the fractions $\frac{x}{y} = \frac{m}{n}$ will be called the **ratio** of the proportion equation.

Exercise 14. Your late fees on a car rental are \$25 a day. You return the car 5 days late.

- Complete the equation:

$$\frac{25 \text{ dollars}}{\boxed{} \text{ days}} = \frac{\boxed{} \text{ dollars}}{5 \text{ days}} \quad (**)$$

- What is the ratio of dollars (in late fees) to days? Give your answer as a reduced fraction.
- Now suppose that you drive the rental car 100 miles every day. Then the total amount driven is proportional to the number of days. Complete the equation:

$$\frac{\text{number of days}}{\text{number of miles driven}} = \frac{5}{\boxed{}} \quad (\dagger)$$

- What is the ratio of days to miles? Give your answer as a reduced fraction.

Exercise 15. A measurement of a worker's financial exploitation by their bosses is the **ratio of exploitation** E , given by³ the formula:

$$E = \frac{\text{amount taken by the bosses}}{\text{amount paid to the worker for labor}}.$$

- If the ratio of exploitation is $E = 1$, and a worker is paid \$253.75 (the amount paid at Georgia's minimum wage for 35 hours of work with no benefits) for their labor, then what's the amount taken by the bosses? (*Hint*: $1 = \frac{1}{1}$.)

- "Boss gets a dollar, I get a dime. . ." In this situation, what's the ratio of exploitation E ?

- Suppose your bosses assign you to work a solo project for a client who is billed \$500 for labor. If the ratio of exploitation is 1, how much do you take home?

³Marx, *Capital*, Vol. 1, Ch. 9, S. 1 ("Both ratios, $\frac{s}{v}$ and $\frac{\text{surplus labour}}{\text{necessary labour}}$, express the same thing in different ways; [. . .] The rate of surplus-value [that is, $\frac{s}{v}$] is therefore an exact expression for the degree of exploitation of labor-power [. . .].")