MATH 1001 Worksheet #2 | Triangles, Polygons, and Circles

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Contents

A. Similar Triangles.

A **triangle** is defined to be three line segments, joined end to end, starting and ending at the same point.

Any triangle can be *described* by:

- its *shape*, and
- its *scale*.

Let us now define "*shape*" and "*scale*" for triangles.

Given a triangle with side lengths

$$a,\ b,\text{ and } c \qquad (\textit{smallest to largest, } a \le b \le c),$$

multiplying all three side lengths by the same number (call it the SCALE) yields a triangle with the **same shape**.

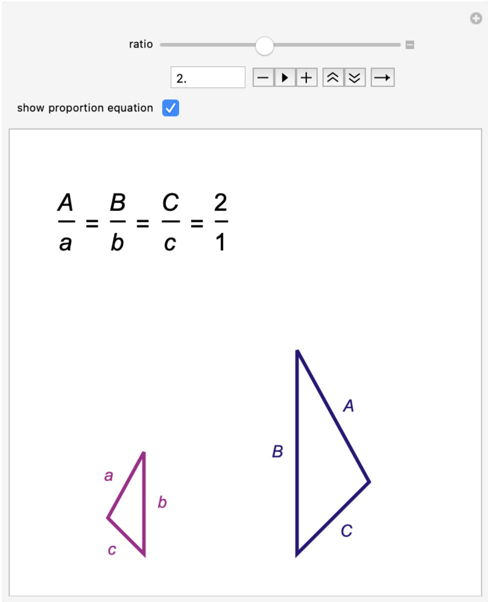
Let us call the second triangle's side lengths

$$A,\ B,\text{ and } C \qquad (\textit{smallest to largest, } A \le B \le C).$$

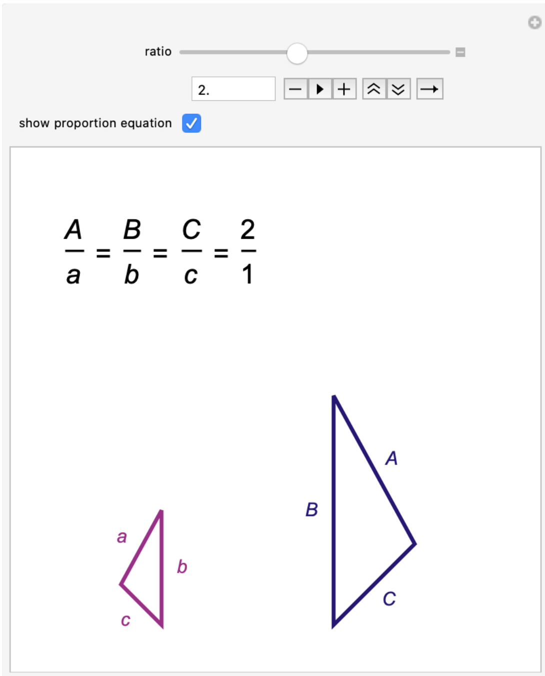
Then

$$\frac{A}{a} = \frac{B}{b} = \frac{C}{c} = \text{the SCALE}$$

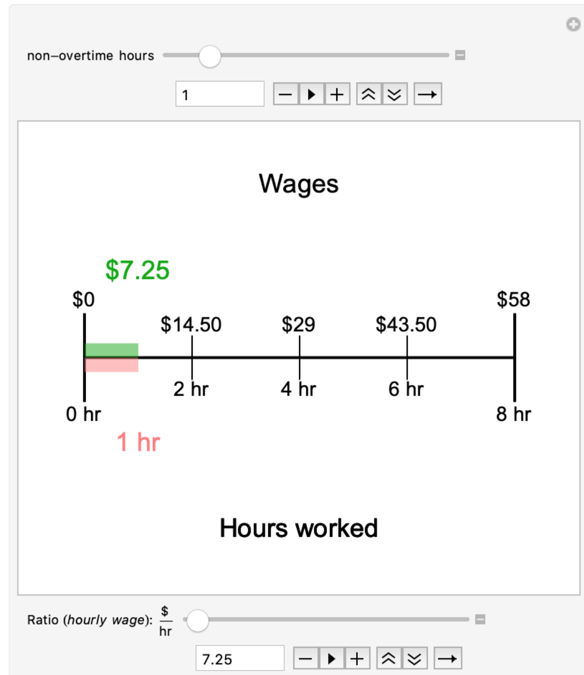
and we say the two triangles are **similar triangles**.



☞ Notice that the SCALE is the *ratio* of the proportion equation $\frac{A}{a} = \frac{B}{b} = \frac{C}{c}$, just as the HOURLY WAGE equals the *ratio* of the proportion equation $\frac{\text{wages paid}}{\text{hours worked}} = \frac{\$14.50}{2 \text{ hr}} = \frac{\$29}{4 \text{ hr}}$.



Ratio $= \frac{2}{1}$



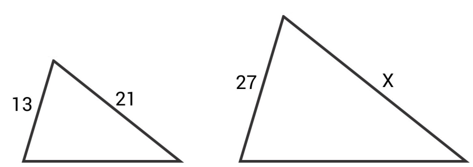
Ratio $= \frac{\$7.25}{1 \text{ hr}}$

To find an unknown side length, given two *similar* triangles, follow these steps:

- For the first triangle, label the smallest side as a, the next smallest side as b, and the largest side as c.
- For the second triangle, label the smallest side as A, the next smallest side as B, and the largest side as C.
- Then by definition of similar triangles,

$$\frac{A}{a} = \frac{B}{b} = \frac{C}{c}.$$

- Fill in the known side lengths to get an equation of two fractions that involves the unknown side length, x.
- Cross-multiply, then find x.



$a = 13$
$b = 21$
$c = ?$

$A = 27$
$B = x$
$C = ?$

$$\frac{27}{13} = \frac{x}{21}$$

$$13x = (27)(21)$$

$$x = \boxed{\frac{567}{13}} \approx \boxed{43.62}$$

Exercise 1. Solve each proportion equation. Round to the nearest hundredth when necessary.

(a) $\frac{1}{1} = \frac{x}{4}$

(b) $\frac{1}{2} = \frac{x}{4}$

(c) $\frac{9}{20} = \frac{x}{60}$

(d) $\frac{100}{17} = \frac{50}{x}$

(e) $\frac{x}{12} = \frac{40}{3}$

(f) $\frac{8}{15} = \frac{x}{20}$

(g) $\frac{8.1}{4.4} = \frac{x}{6.4}$

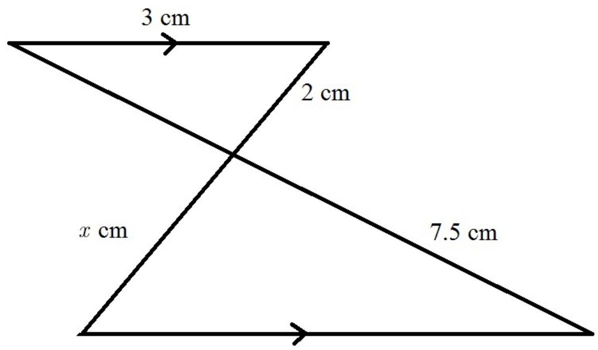
(h) $\frac{10.2}{x} = \frac{21}{30}$

(i) $\frac{2}{3} = \frac{17}{x}$

☞ Cross-multiplying works for *all* these problems.

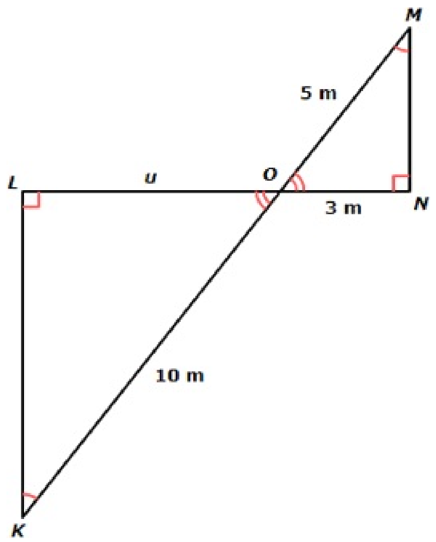
Exercise 2.

(a) Find x.



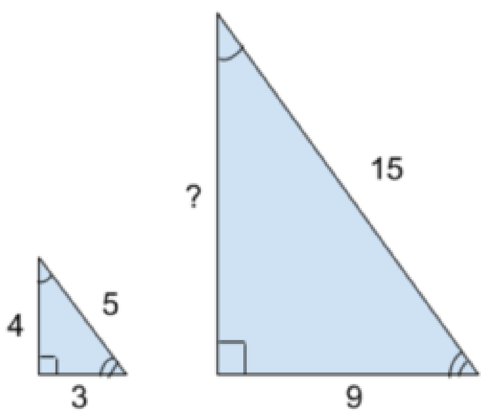
Here, cm means *centimeters*.

(b) Find u.



Here, m means *meters*.

(c) Find $?$.



B. Perimeter, Area, and Circumference.

A **polygon** is a sequence of line segments joined end to end

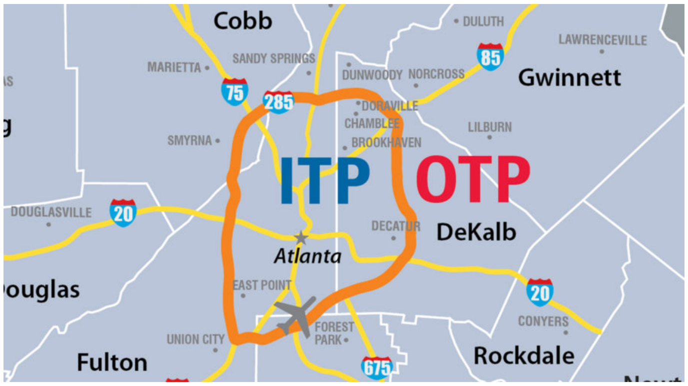
- which starts and ends at the same point,
- such that the line segments meet only at their endpoints (that is, they do not "cross").

(B1) Perimeter of a Polygon

Etymologically, the word PERIMETER means, "*distance around*."

Interstate 285 is called "the perimeter" because it "goes around" Atlanta.

Note: in math, the word "perimeter" always refers to a POLYGON (that is, not to a *curved* shape, like a circle—nor, technically, to the curved road I-285).



To find the **perimeter** of any polygon, add up all its side lengths.

Exercise 3. Draw a picture of a rectangle 300 ft. long and 125 ft. wide. Then calculate its perimeter.

Exercise 4. Suppose a triangle has side lengths 5, 10, and 7. What's its perimeter?

(B2) Area of a Rectangle

The **area** of a two-dimensional shape (like a rectangle, or a triangle, or a circle) is the amount of two-dimensional space inside it.

Exercise 5. The area A of a rectangle with length ℓ and width w is given by the following formula:

$$A = \ell \cdot w$$

The units that measure area are SQUARE UNITS. For example, if a room's dimensions are given in FEET (ft.), then the area is given in SQUARE FEET (sq. ft. or ft.^{2}).

- Find the area of the bedroom pictured below (12 feet on all sides). Don't forget the units!
- Find the area of the living room pictured below (12 feet long, 15 feet wide).

Do you see anything *misleading* about this realtor's floorplan?

Exercise 6. Draw a square with side length 16 inches (in.). Then find its area.

Exercise 7. Divide the square you drew in Exercise 6 in half with a diagonal line. What's the area of each triangle?

Exercise 8. Suppose that a rectangular bedroom is 12 feet long and 10 feet broad.

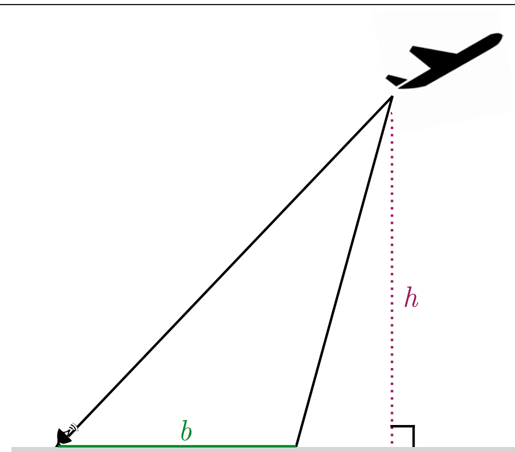
- What's the area of the bedroom?
- An 1865 report by the British government stated that a rented one-bedroom house in Bedfordshire County, England housed a working-class married couple and six children.[1] Suppose all eight residents shared a single $12' \times 10'$ bedroom. If they equally divided the bedroom's floor-space, how much floor-space did each person get?
- (*Challenge:*) Assume the ceiling in the bedroom was $5\frac{1}{2}'$ ft. tall. How much *three-dimensional space* (that is, how much *volume*, in *cubic* feet) did each person get?
- An engaging exploration of how many humans can *comfortably* occupy a given amount of space can be found at this link: "7.3 Billion People, One Building"

(B3) Area of a Triangle

The formula for the area of a triangle is

$$A = \frac{1}{2}bh$$

where h is the triangle's height and b is the length of its base.



Exercise 9. Round to the nearest tenth, if needed.

(a) Find the area of a triangle with $b = 20$ in. and $h = 9$ in.

(b) Find the area of a triangle with $b = 7$ cm and $h = 11$ cm.

[1]Parliament public health report cited in Marx, *Capital*, Vol. 1, Ch. 25, S. 5(e)

(B4) Area vs. Perimeter

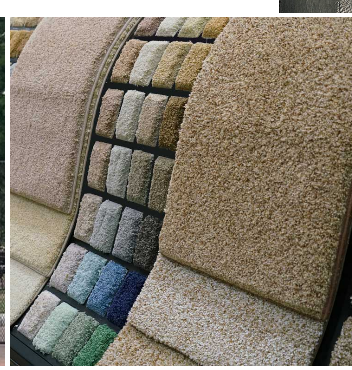
Exercise 10. Perimeter or area?

(a) A fence around the border of a plot of land

(b) New carpet for a bedroom

(c) Paint for a bedroom wall

(d) Wood molding for a picture frame



(B5) Area and Circumference of Circles

The **area** A **of a circle** is given by the formula

$$A = \pi r^2$$

where r is the radius.

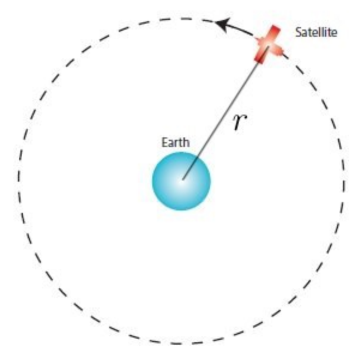
☞ Notice the "squared" in the formula for area, which is measured in SQUARE UNITS.

The **circumference** C **of a circle** is given by the following formula:

$$C = 2\pi r$$

The circumference is the "distance around" the circle.

For example, if a kid swings a ball on a string in a circle (*image at right*), then the distance traveled by the ball in one complete revolution is the circumference.



☞ An alternate version of the formula for circumference is

$$C = \pi d$$

where $d = 2r$ is the circle's diameter.

Exercise 11.

(a) Find the area and circumference of a circle with radius 16 in.

(b) Find the area and circumference of a circle with diameter 12 cm.

(c) Find the area and circumference of a circle with radius 25 miles (mi.).