MATH 1001 Worksheet #3 | The Rectangular Coordinate System

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Contents

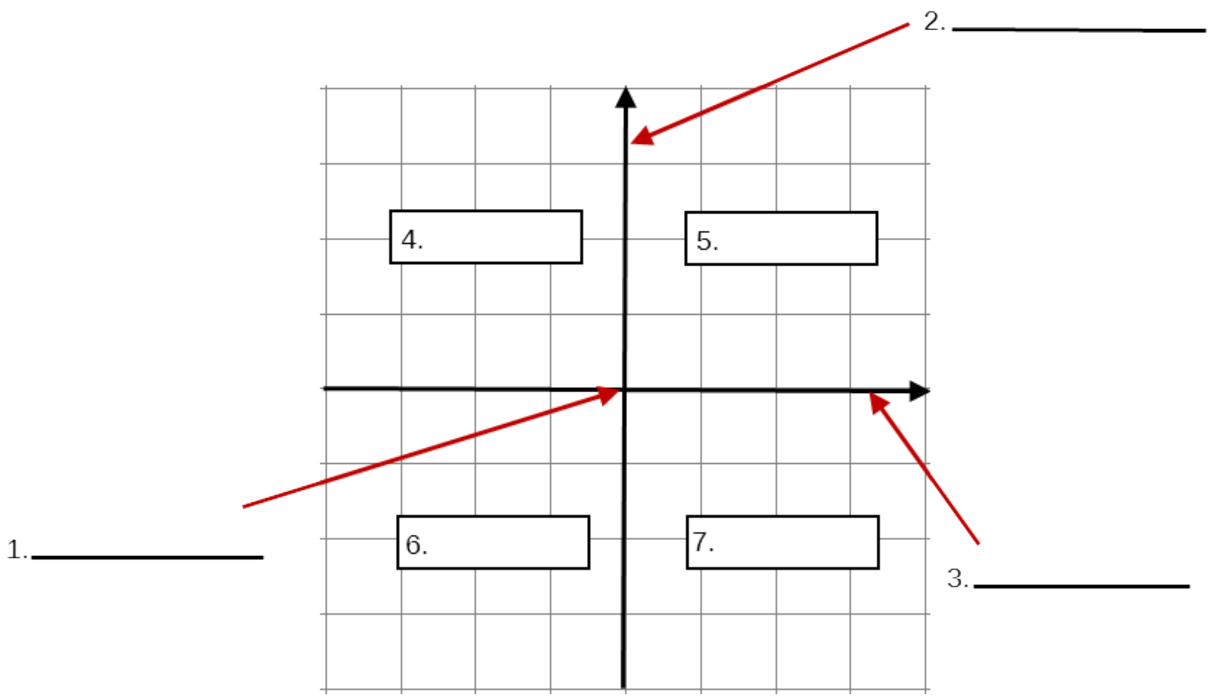
A. The Rectangular Coordinate System.

(A1) Vocabulary

The picture below is called the **rectangular coordinate system**.

Exercise 1. Label the parts of the rectangular coordinate system with the appropriate term from the following list:

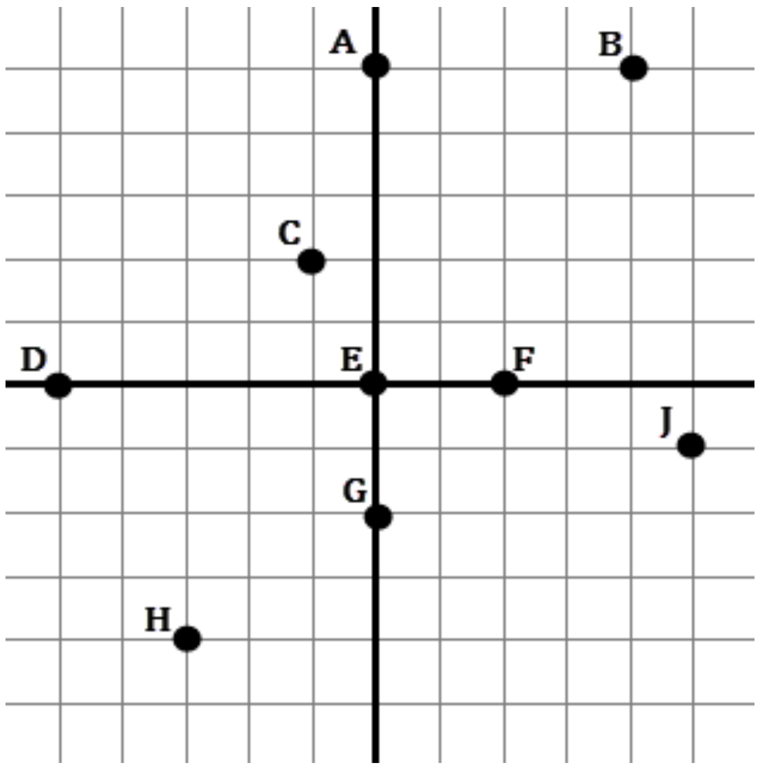
- Quadrant I
- Quadrant II
- Quadrant III
- Quadrant IV
- x-axis
- y-axis
- origin



(A2) Coordinates of a Point in the Rectangular Coordinate System

Exercise 2. Write the coordinates for each point shown as an ordered pair $(\square, \square)$.

Assume all grid-lines are spaced 1 unit apart.



$A = \Big(\quad , \quad \Big)$

$B = \Big(\quad , \quad \Big)$

$C = \Big(\quad , \quad \Big)$

$D = \Big(\quad , \quad \Big)$

$E = \Big(\quad , \quad \Big)$

$F = \Big(\quad , \quad \Big)$

$G = \Big(\quad , \quad \Big)$

$H = \Big(\quad , \quad \Big)$

$J = \Big(\quad , \quad \Big)$

We can use a rectangular coordinate system to represent any relationship between two variable quantities.

Exercise 3. SUPPOSE YOU WORK A FOUR-HOUR SHIFT paid at $\$10$/hr. At the end of today's shift, your boss gives you a choice about how long your next shift will be.

- In the space provided below, draw a rectangular coordinate system (x-axis and y-axis).
- Label the x-axis (on the right end) as:

  $$x = \text{increase in work-hours}$$

- Label the y-axis (on the top end) as:

  $$y = \text{total pay for your next shift}$$

- Mark the x-axis with "tick marks" going up by 1 each, from -4 at the far left, to 4 at the far right.
- Mark the y-axis with tick marks going up by $\$10$ each, with a maximum of $\$100$.
- IF YOU MAKE NO CHANGE, and stick with a four-hour shift, you'll get paid how much money for your next shift? (Zero increase in work-hours means your next shift will be four hours, so you get 4 x \$10 $= \$40$.) Indicate this by plotting the point whose coordinates are:

  $$(0, 40)$$

  Label the point by writing this ordered pair next to it.
- IF YOU WORK 1 EXTRA HOUR, you'll take home how much for your next shift? ($\$50$.) Add this information to the picture by plotting another point and labeling it with its coordinates.
- IF YOU WORK 1 HOUR LESS—in other words, *an "increase" of negative 1*—you'll take home $\$30$. Indicate this fact by plotting and labeling a point that has $x = -1$.
- How much will you get paid if you work an EIGHT-HOUR SHIFT? Plot and label one more point that represents your answer to this question.
- *Don't connect the dots!*

The picture you've drawn is an example of what we call a **scatter plot**. In a scatter plot, we see individual points, each of which represents a different observation or fact ("$x = 1$ more hour means $y = \$50$ pay for the shift").

B. Graphing a Line from its Equation.

Exercise 4.

There are about 50 weeks in a year. If we assume a 40-hour working week, we find that there are about

$$50 \text{ wk} \times 40 \, \frac{\text{working hours}}{\text{wk}} = 2000 \text{ working hours in a year.}$$

We can use this estimate to create a model that predicts the first year of income from a job that pays a signing bonus of $\$3000$ and hourly pay of $\$x$ / hr:

$$\boxed{\text{total pay for the first year} \;=\; \text{signing bonus} \;+\; 2000 \cdot \left(x \, \frac{\text{dollars}}{\text{hr}} \right)}$$

That is,

$$y = 3000 + 2000x$$

where

y is total pay for the first year, and
x is hourly pay.

- How much would you make in your first year at $\$25$ per hour?
- How much would you make in your first year at $\$30$ per hour?
- Draw a rectangular coordinate system.
- Label the x-axis as "$x =$ hourly pay".
- Label the y-axis as "$y =$ pay for the year".
- Mark ticks on the two axes appropriately.
- Plot the two points that represent your answers to the first two bulleted parts of this exercise.
- The graph of this model is a STRAIGHT LINE. Join the two points you plotted in the previous part, and extend to a line.

The picture you drew in this exercise is an example of the **graph of an equation—**in this case, the graph of $y = 3000 + 2000x$.

C. Solving an Equation for One of its Variables.

The graph of an equation in TWO variables is drawn in two dimensions (the x-axis and the y-axis).

Each dimension is represented algebraically by a different variable, which appears visually on a different axis.

In this class, we'll sometimes see equations with THREE OR MORE variables. We'll see some on this page.

Exercise 5.

The equation[1]

$$S = E \cdot Pn$$

is a mathematical model of the relationship between

- $n =$ the number of workers at a factory or shop,
- $P =$ the daily pay of each worker,
- $S =$ the bosses' daily take-home, and
- $E =$ the ratio of exploitation.[2]

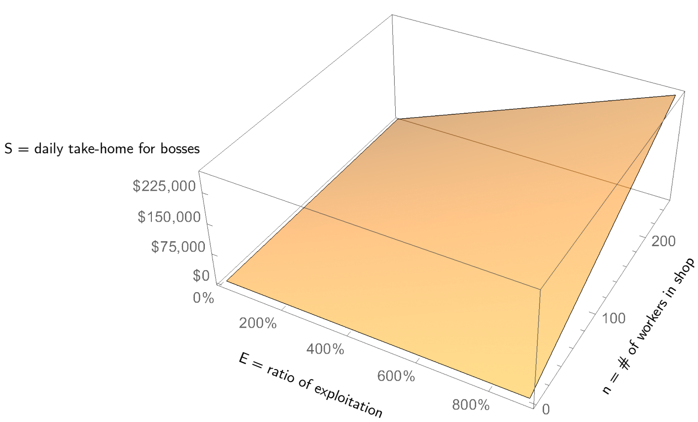
Let's assume each worker gets paid

$$P = \frac{\$15}{\text{hr}} \times \frac{8 \text{ hr labor}}{\text{working day}} = \$120 \text{ per working day.}$$

Substituting for P gives us:

$$S = 120E \cdot n$$

We used a computer to graph this equation in three variables. The result is a three-dimensional picture:



☞ You will NOT be tested on three-dimensional graphs!

However, you *are* expected to be able to complete the following problems:

- If $E = 1$, $P = 120$, and $n = 50$, what is S?

- Solve the equation $S = E \cdot Pn$ for E.

  (This means, "Get E by itself on one side of the equation.")

- Solve the equation $S = E \cdot Pn$ for P.

[1] Adapted from Marx, *Capital*, Vol. 1, Ch. 11.

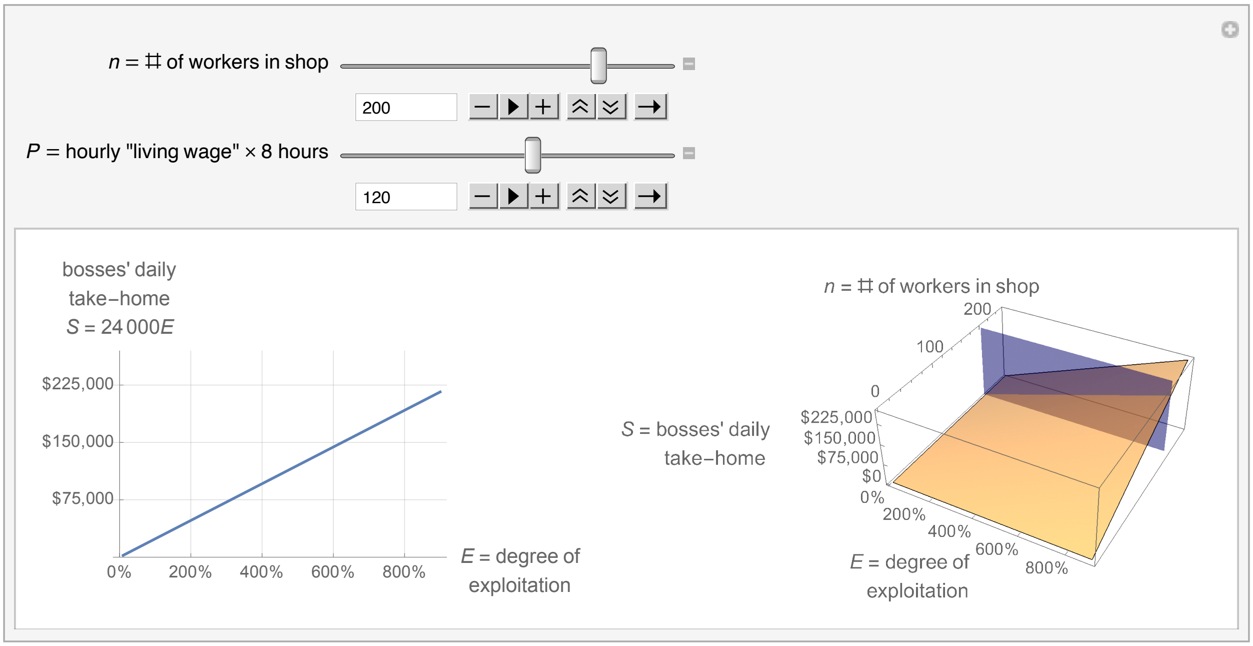
[2] See Worksheet 1 for a definition of the ratio of exploitation—but you don't need it to work this exercise, and you *certainly* won't be tested on it.

D. Finding the y-Intercept and Slope of the Graph of a Line.

Exercise 6.

In the applet screenshotted below, we took $n = \#$ of workers $= 200$, which gave us the equation

$$S = 24,000E.$$



- What is the y-intercept on the (*above left*) graph of $S = 24,000E$? Use algebra to give an exact answer.

  (We can always *estimate* where the line meets the y-axis just by "eyeballing" the graph. But in this case, the very large scale on the y-axis means that our estimate wouldn't be very precise.)

- In plain English, explain how you found the y-intercept using algebra.

- Interpret what the y-intercept means by filling in the blanks:

  The bosses' daily take-home is ____________ when $E =$ _____.

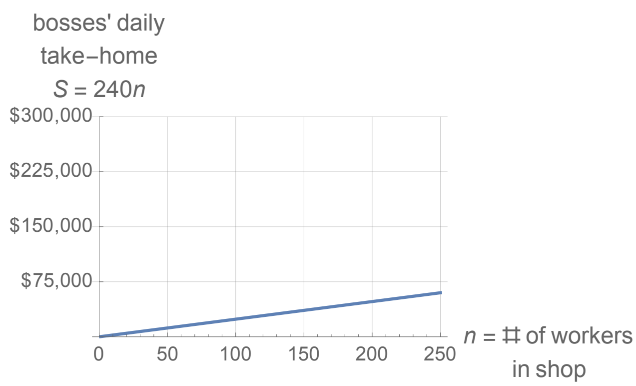
- In the *above left* graph, what is E when $S \approx \$150,000$?

In these last two parts of Exercise 6, we've taken $P = 120$ and experimented with different values of E.

- At $E = 200\%$, the model becomes

$$S = 240n$$

and the graph is:

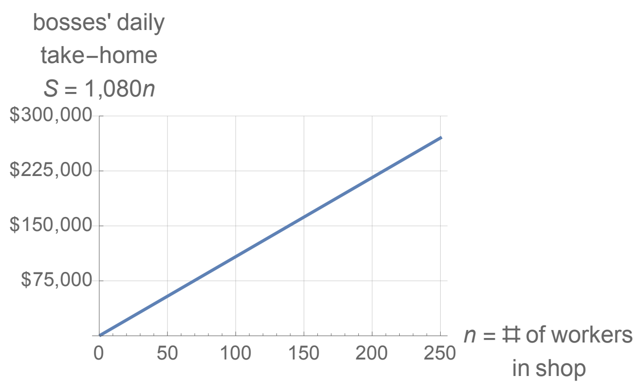


Find S when $n = 150$ and when $n = 50$. Then find the slope of the line.

- At $E = 900\%$, the model becomes

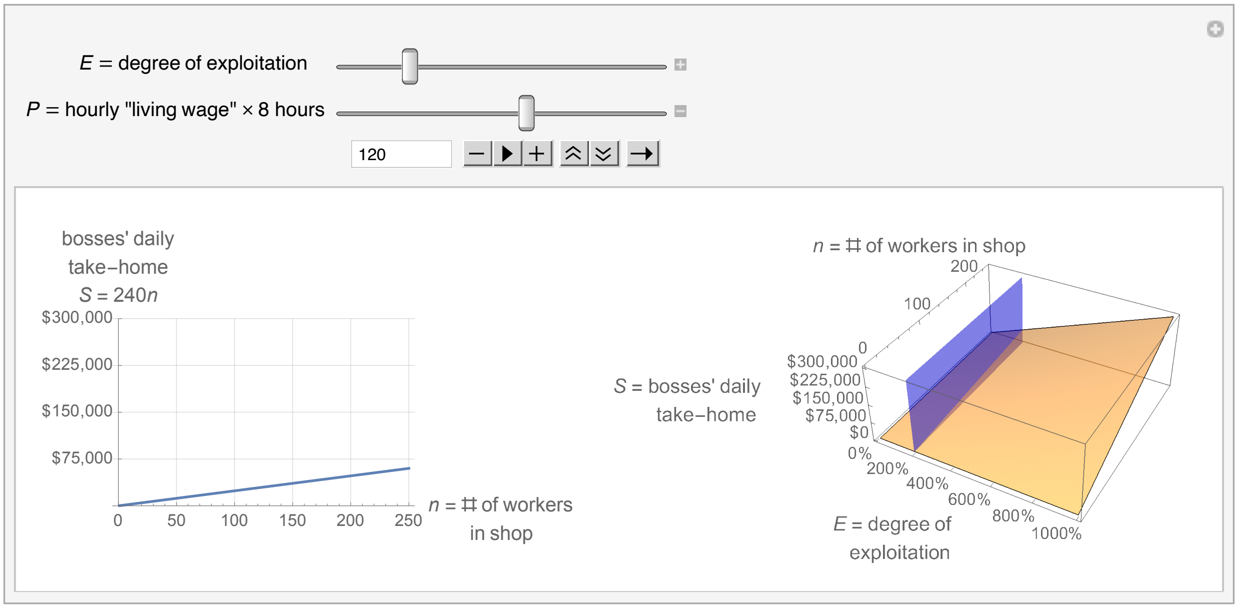
$$S = 1080n$$

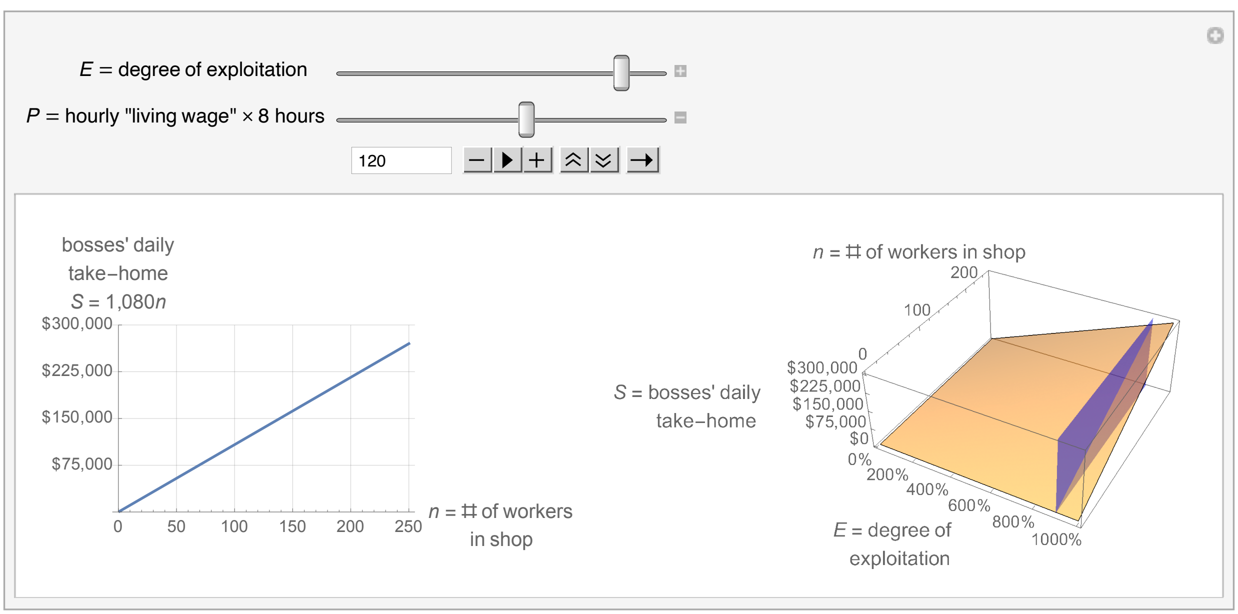
and the graph is:



Find S when $n = 150$ and when $n = 50$. Then find the slope of the line.

The next and last page of this Worksheet shows the three-dimensional pictures corresponding to the graphs on this page. You are free to ignore them! They're just included in case technical issues prevent the instructor from showing you the applet and/or letting students play with it during class.


E = degree of exploitation
P = hourly "living wage" × 8 hours
120
bosses' daily take–home
S = 240n
$300,000
$225,000
$150,000
$75,000
0
50
100
150
200
250
n = ⌗ of workers in shop
n = ⌗ of workers in shop
200
100
0
S = bosses' daily take–home
$300,000
$225,000
$150,000
$75,000
$0
0%
200%
400%
600%
800%
1000%
E = degree of exploitation


E = degree of exploitation
P = hourly "living wage" × 8 hours
120
bosses' daily take–home
S = 1,080n
$300,000
$225,000
$150,000
$75,000
0
50
100
150
200
250
n = ⌗ of workers in shop
n = ⌗ of workers in shop
200
100
0
S = bosses' daily take–home
$300,000
$225,000
$150,000
$75,000
$0
0%
200%
400%
600%
800%
1000%
E = degree of exploitation