MATH 1001 Worksheet #6 | Financial Formulas

Last revised: 2021-06-25 14:00

Contents

A. Vocabulary for Financial Formulas.

Typically, students' three biggest challenges with the material covered in this Worksheet are:

- Choosing the right formula,
- Identifying which number goes with which variable, and
- Entering the formula into the calculator.

Definitions:

principal	*1.* the amount of money initially advanced in a loan or investment *2.* the account balance based on which interest is calculated
to compound	(*verb*) to calculate interest, and then add it to the principal
simple interest financing	interest is *never* compounded—the interest is based only on the initial advance
compound interest financing	interest is compounded *regularly*—say, some number, n, of times each year
installment loan	loan paid back in regular "installment" payments (most often monthly)
car loan	installment loan for a car
mortgage	installment loan for a house
closing	the time at which a house loan (or **mortgage**) begins
points at closing	a fee paid to the lender up-front—that is, "at closing"
down payment	a *portion of the house's price* paid up-front, "at closing"

Financial formulas (Chapter 8)

§8.3	Simple Interest.	$I = Prt$
§8.3	Future Value.	$A = P + I$
§8.3	Future Value for Simple Interest.	$A = P \cdot (1 + rt)$
§8.4	Future Value for Compound Interest compounded n times each year.	$A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}$
§8.4	Present Value for Compound Interest compounded n times each year.	$P = \dfrac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$
§8.4	Future Value for Continuously Compounded Compound Interest.	$A = Pe^{rt}$
§8.5	Future value of an annuity/IRA compounded n times each year.	$A = \dfrac{\text{PMT} \cdot \left[\left(1 + \dfrac{r}{n}\right)^{nt} - 1\right]}{\dfrac{r}{n}}$
§8.5	Regular payments needed to reach a desired future value.	$\text{PMT} = \dfrac{A \cdot \dfrac{r}{n}}{\left[\left(1 + \dfrac{r}{n}\right)^{nt} - 1\right]}$
§8.6	Car loan and mortgage payments.	$\text{PMT} = \dfrac{P \cdot \dfrac{r}{n}}{\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}$
§8.7	Down payment of $D\%$.	$D\% \cdot$ (sale price)
§8.7	x points at closing.	$x\% \cdot$ (sale price)
§8.7	Amount of mortgage.	(sale price) − (down payment)

B. Compound Interest.

To compound interest means, to calculate interest, and then add it to the principal.

The principal thus grows larger and larger each time interest is compounded.

For example, suppose a bank loans out $\$1000$, at 10% annual interest, compounded ONCE a year.

- After 1 year, the interest is $\$1000 \times 10\% = \$100$. This $\$100$ is added to the principal.
- Now the principal is $\$1100$. At the end of the second year, interest is compounded with the new principal: $\$1100 \times 10\% = \$110$.
- The third year begins with a principal of $\$1110 + \$110 = \$1220$, and ends with another $\$1220 \times 10\% = \$122$ being added to the principal.
- Each year, the amount of increase in future value gets larger and larger.

☞ *For compound interest, future value grows exponentially.*

☞ The rate of compound interest is given as a percentage, the **APR** (annual percentage rate—see Worksheet #5).

For credit cards, savings accounts, and student loans, we have the following two formulas:

- Given the *principal*, the FUTURE VALUE after t YEARS is: $\boxed{A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}}$
  - $P =$ principal (account balance)
  - $r =$ APR (annual percentage rate of interest)
  - $n = \#$ of times per year interest is "compounded" (calculated and added to the principal)
  - $t =$ time IN YEARS
- Given the *future value*, the PRESENT VALUE is: $\boxed{P = \dfrac{A}{\left(1 + \frac{r}{n}\right)^{nt}}}$

Exercise 1. Theo's credit card has an APR of 24%. Interest is compounded monthly.

(a) What rate of interest is Theo charged *per month*?

(b) If Theo borrows $\$100$ and waits two months to pay it back, plus interest, how much will Theo have to pay back in total?

(c) How much of that is interest (profit to the credit card company)?

Exercise 2. *(Optional.)* Suppose compound interest is compounded once per year. Substitute the appropriate value of n, and rewrite each of the formulas $\boxed{A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}}$ and $\boxed{P = \dfrac{A}{\left(1 + \frac{r}{n}\right)^{nt}}}$ more simply.

Exercise 3. A bank advertises a "high-yield" savings account with an APR of 0.40%, compounded monthly.

(a) Kenzie deposits $\$1,000$ in such an account and waits 10 years for interest to accumulate. What will the account balance be?

Answer: $\boxed{\$1040.80}$

(b) How much of that is interest?

(c) What percentage of the principal is the interest?

Answer: $\boxed{\text{About } 4.1\%}$

Exercise 4. Darius wants to accumulate $\$1,100,000$ using a financial instrument that earns an APR of 5%, compounded quarterly.

(a) How much would Darius need to deposit today in order to reach his financial goal in 5 years?

ANSWER: $\boxed{\$858,009.40}$

(b) How much would Darius need to deposit today in order to reach his financial goal in 40 years?

ANSWER: $\boxed{\$150,725.80}$

C. Annuities/IRA's.

An **annuity** is an investment account available to consumers at many banks.

☞ An **IRA** is a type of tax-free annuity available at many banks.

In this course, we'll assume for all annuity/IRA problems that:

- the consumer makes a fixed payment n times each year, and
- the lender compounds interest the same number of times each year.

The two formulas we need for annuities/IRA's are:

- Given the *principal*, the FUTURE VALUE after t YEARS is: $\boxed{A = \dfrac{P \cdot \left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\frac{r}{n}}}$

  - $P =$ principal (account balance)
  - $r =$ APR (annual percentage rate of interest)
  - $n = \#$ of payments each year
  - $t =$ time IN YEARS

- Given the desired *future value* (the *financial goal*), the PAYMENT is: $\boxed{\text{PMT} = \dfrac{A \cdot \frac{r}{n}}{\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}}$

Exercise 5. Kai's insurance doesn't cover the $\$9,000$ surgery he needs, so he'll need to finance it. He considers the following two options:

CREDIT CARD: CareCredit, APR 26.99%

ANNUITY: Georgia's Own Credit Union,[1] APR 2.9%

Both accounts compound interest *monthly*.

(a) Suppose Kai borrows the $\$9,000$ using the CareCredit credit card, and never makes a single payment. How much will he owe after 5 years? (For this type of problem, use the future value formula for interest compounded n times a year.)

ANSWER: $\boxed{\$34,184.49}$

(b) If Kai waits 5 years for the surgery, and uses the annuity to pay for it, what will be the monthly payment?

ANSWER: $\boxed{\$139.47}$

(c) What would Kai's *total payments* be, with the annuity?

ANSWER: $\boxed{\$8374.11}$

(d) How much *interest* did the annuity pay to Kai?

ANSWER: $\boxed{\$625.86}$

(e) What should Kai do?

[1] As of Jun. 25, 2021, Georgia's Own Credit Union advertises annuities at an APR between 2.30% and 3.50%.

Exercise 6. At age 20, Ami decides to save $\$1,500,000$ over the course of the next 40 years in hopes of retiring at age 60.

(a) If she saves for retirement using an IRA with an APR of 5%, what will her monthly payment be?

ANSWER: $\boxed{\$982.95}$

(b) If she saves for retirement using an IRA with an APR of 3%, what will her monthly payment be?

ANSWER: $\boxed{\$1619.77}$

D. Installment loans.

Large purchases like cars and houses can be paid for using an **installment loan**.

- As long as the consumer keeps making payments, they are allowed to use the car or house.
- At the end of the loan, the consumer becomes the owner of the property.

In this course, we'll assume for all car loan and "house loan" (mortgage) problems that:

- the consumer makes a fixed payment n times each year, and
- the lender compounds interest the same number of times each year.

We have one formula for installment loans (used both for car loans and mortgages):

- Given the *amount of the loan*, the PAYMENT is: $$\boxed{\text{PMT} = \frac{P \cdot \frac{r}{n}}{\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}}$$

Exercise 7. Find the monthly payment for a 5-year car loan for $\$10,000$ at an APR of 6%.

ANSWER: $\boxed{\$193.33}$

E. Mortgages.

A **mortgage** is an installment loan used to pay for a house.

You are expected to know the following definitions for mortgages:

- **Closing** refers to the time at which a house loan (or **mortgage**) begins—that is, when the paperwork for the loan is finalized.
- **Points at closing** refers to a fee paid to the lender up-front—that is, "at closing."
- The **down payment** is a *portion of the house's price* paid up-front, "at closing."

The following steps are suggested for mortgage problems *before* calculating the monthly payment:

1. Find the points at closing, after converting the given number of points—x, let's say—to the *percentage* $x\%$:

$$x \text{ points at closing} = x\% \times \text{house's sale price}$$

2. Find the down payment, which is given as a PERCENTAGE—call it $D\%$:

$$\text{down payment} = D\% \times \text{house's sale price}$$

3. Find the REMAINING AMOUNT THAT MUST BE BORROWED—this is called the **amount of the mortgage**:

$$\begin{array}{rl} & \text{house's sale price} \\ - & \text{down payment} \\ \hline & \text{amount of mortgage} \end{array}$$

Exercise 8. The price of a home is $\$210,000$. The bank requires a 15% down payment and two points at closing. The cost of the home is financed with a 30-year mortgage at 6.5%.

Find the down payment, the amount of the mortgage, and the monthly payment. Round to the nearest dollar.

Formula for installment loan payments: $\text{PMT} = \dfrac{P\left(\frac{r}{n}\right)}{\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}$

Points at closing:

Down payment:

Mortgage amount:

Monthly payment: