Profit and power

Notes on Capital, Vol. 1-4

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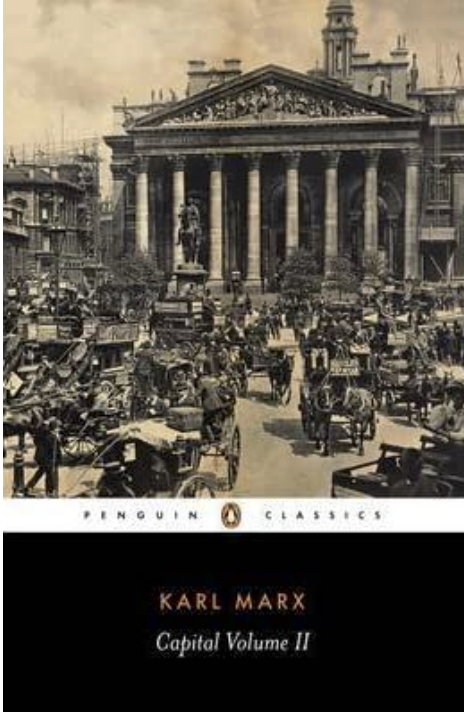Julie La Corte

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Contents

1 Introduction (Discrete flows and surplus)

Let us begin at a very general level of abstraction.

Whatever is happening in Marx's economic model, happens *over time*. The motion can be frozen and a snapshot examined; then relations can be identified and diagrammed. The nodes thus related may represent commodities, individuals, classes, spheres of production, and so on. But even when time is suspended for the purposes of exposition, this "space" of frozen social and economic relationships must be understood as being embedded in the context of some determinate historical moment, with some definite average level of technology, at some identifiable stage of some particular economic process.

To illustrate the point, suppose the flows of capital are all frozen in place. What appears is a bewildering jumble of bodies, commodities, farms, factories, bank notes, metal coins, and so on. Capital itself disappears, in the sense that capital (in its monetary form) is "money that breeds money"— which it cannot possibly do so long as time is held fixed. If we assume that all transactions are fair, as Marx does, then at the instant of exchange, without additional information about what came before or is yet to come, we cannot even tell whether the buyer or the seller stands to gain, since the money and goods exchanged are required to be equal in value.

By studying a *sequence* of transactions, on the other hand, we can follow the "money" as capital is converted into commodity form, reconverted into cash, and so on. At any given instant, the value of a mass of capital (sum of money, quantity of goods, etc.) must appear in some qualitative form. Certain transactions change the value's qualitative form. For instance, from the baker's perspective, the value of a loaf of bread changes in form from commodity to money when the bread is sold:

$$\underset{\text{held by baker before sale}}{1 \text{ loaf bread}} \quad \longleftrightarrow \quad \underset{\text{held by baker after sale}}{2 \text{ pennies}}$$

There are also processes that change the value purely quantitatively. Human labor is such a process, transforming raw materials into more valuable finished goods through some concrete process of production, e.g. spinning cotton into yarn:

$$\underbrace{10 \text{ lb. cotton} + \tfrac{1}{4} \text{ spindle}}_{\text{worth } 12 \text{ shillings}} + \underbrace{6 \text{ hr spinning}}_{\text{worth } 3 \text{ shillings}} \quad \rightsquigarrow \quad \underbrace{10 \text{ lb. yarn}}_{\text{worth } 15 \text{ shillings}}$$

These changes in quality and quantity are ordered in time and finite in number. We'll call such a sequence of changes a **discrete flow**, whether it represents a sequence of exchanges $\longleftrightarrow$, applications of labor $\rightsquigarrow$, or some combination thereof.

At the micro level, an individual mass of value undergoes a discrete flow, alternating between monetary form and commodity form.

At the macro level, the *total* mass of value extant in a society at a given time and place undergoes a discrete flow. Parts of the total mass are in commodity form while other parts are in monetary form. Parts of the total mass differ in function, as well: one portion is tied up in production, another portion circulates, another portion is hoarded, etc. Marx decomposes the total mass of value into **components**, each of which aggregates the portion of the whole which is in this or that qualitative form and employed in such and such an economic function. The analysis of the discrete flow of the total mass of value concerns both the flow of the entire mass and the flows of its components.

So far we have outlined:

- **discrete microflows** of individual elements of capital in monetary/commodity/etc. form,
- **discrete macroflows** of masses of capital decomposed into formal and/or functional components, and
- a common axis of time.

The functional component that holds the greatest importance for the capitalist is **surplus**, excess beyond one's needs.

The flow of surplus appears at every scale in *Capital*, most obviously in production ($c + v \rightsquigarrow c + v + s$, see §4.1) and in crises of overproduction, but also at the microlevel of transactions governed by a given value-form (§2.5.1), and again at the aggregate level of economic classes as the material expression of *force*, e.g. macroflows of profit which are the material expression of the collective capitalist's drive to accumulate, *by force as necessary*. We will elaborate on the following points in the sections to follow:

- At the transactional level, the *relative form* (Definition 2.4) represents personal surplus, that which *must* be exchanged for an equivalent in order to realize its worth. A *value-form* (Definition 2.9) encodes each kind of commodity (Definition 2.9) as either a relative or equivalent form. We formalize this and give a geometric representation (Definition 2.12) of a given value-form as a diagram made up of pointed arrows (i.e. a directed graph) such that each arrow links a kind of commodity that *must* be exchanged, **i**, say, to a a different kind of commodity, call it **j**, which **i** exchanges for. We label each arrow by the proportion x/y in which the two respective kinds of commodity **i** and **j** exchange:
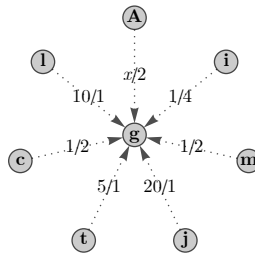
  A sequence (microflow) of $n - 1$ transactions

  $$x_1 \mathbf{a}_1 \longleftrightarrow x_2 \mathbf{a}_2 \longleftrightarrow \cdots \longleftrightarrow x_n \mathbf{a}_n$$

  can then be represented as a path $\left(\mathbf{a}_k\right)_{k=1}^{n}$ in the diagram (ignoring directions of arrows) together with a sequence $\left(x_k\right)_{k=1}^{n}$ of the masses $x_k > 0$ of $\mathbf{a}_k$ exchanged.

- At the industrial level, surplus motivates production. The acquisition of surplus, $c + v \rightsquigarrow c + v + s$, is the industrial capitalist's sole motive qua capitalist: this fact is expressed in the principle, advanced by Malthus and repeatedly cited by Marx in *Capital*, that the capitalist expects a *profit* (= mass of surplus = increment, see §5) on all parts of the capital advanced.

- At the macroeconomic level, the aggregate of all surpluses of value, once such surpluses have been produced by various concrete types of industrial labor, are redistributed to the various industries ("spheres of production") in accordance with a general rate of profit (§5) which is driven to equilibrium by competition for investment capital and subsequently arises as an observable property of markets—although whether it is *actually* observed depends on how the books were kept (that is, the general rate of profit is observable only to the extent that the rates of profit $s_i/(c_i + v_i)$ for each industry i can be calculated based on public information).

- At the level of classes, surplus-seeking behavior appears as a historical force. The conceptual transformation of a society from one in which unmediated use-value (utility) governs all exchange to one governed by the exchange of commodities corresponds with the historical emergence of private property, the legal means of securing all *future* surpluses arising from a farm, mine, factory, laborer, etc. Original accumulation and enclosure (C1, Part 8), chattel slavery, proletarianization, and colonial empire are primary examples of surplus-seeking collective (class) behavior.

In so far as surplus is an explanatory variable that reliably predicts the individual or collective capitalist's actions, the laws of the capitalist mode of production are necessarily laws of surplus-seeking behavior, and deviation from these laws occurs only by chance.

If we accept the premise that surplus is explanatory, then marginal demand plays the role of statistical error, literally denoting deviation from a mean governed by surplus-value[1] s and its relations with other economic variables, e.g. $C = c + v$ and $\theta = c/v$. With respect to the flow of surplus, the impact of marginal demand on exchange is merely circumstantial or "accidental", i.e. is not the necessary result of any law—this statement cannot mean much to the bourgeois economist who believes money is real and sets aside use-value in all their calculations, but must be transparent to the Marxian thinker, for whom labor-power (life itself, flesh and bone, blood and nerve, etc.) is primary and cash is not king.

We see in Marx's model of the capitalist mode of production the unmistakable functional hallmarks of what Foucault a century later would call *bio-power* when speaking of Marx's own historical circumstances:[2] the mechanization of life, e.g. the "anatomo-politics" of labor conditions, and the institution of policies of selective population control which a Marxist might identify with *social reproduction*. Marx's explanation for these phenomena—namely, that a class of haves ensures the reproduction of a class of have-nots who are marked for discipline by birth—differs

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[1] *Surplus cannot be measured in money* except in so far as it has first been transformed into prices; but this transformation is determined by *value* as measured by units of labor-power, and formally can be obviated by expressing all surplus in terms of its untransformed value.

[2] 'concrete arrangements that constitute the great technology of power of the 19th century [...] This **bio-power** was certainly an indispensable element of the development of capitalism; the latter would not have been possible without the controlled insertion of bodies into the machinery of production and the adaptation of [...the] population to economic processes. But this'—the "*ajustement*" of the workers to capitalism—'was not all that was needed; it also required [...] the development of great instruments of the State, as *institutions* of power, [...and] the rudiments of anatomo- and bio-politics, invented in the 18th century as *techniques* of power present at every level of the social body [...]' ◇◇◇ '*agencements concrets qui constitueront la grande technologie du pouvoir au* XIXe *siècle [...] Ce bio-pouvoir a été, à n'en pas douter, un élément indispensable au développement du capitalisme; celui-ci n'a pu être assuré qu'au prix de l'insertion contrôlée des corps dans l'appareil de production et moyennant un ajustement [...] de population aux processus économiques. Mais il a exigé avantage; il lui a fallu la croissance des uns et des autres; [...] le dèveloppement des grands appareils d'État, comme* institutions *de pouvoir, [...et] les rudiments d'anatomo- et de bio-politique, inventès au* XVIIIe *siècle comme* techniques *de pouvoir prèsentes à tous les niveaux du corps social [...]' Histoire de la sexualité, 1: La volonté de savoir,* 185; cf. Hurley Eng. tr., 140-141.

from Foucault's;[3] in *Abnormal*, Foucault explicitly rejects *class* as an adequate explanatory principle.[4] But the response variable, the literal *qualities of life* in a given place and time, is the same for both authors, who are, after all, studying overlapping *durées* in history. Both authors seek to explain how it is that a definite historical implementation of bio-power, a determinate *empirical configuration of social relations* (= input), conceiveable in broad terms as a coercive regime of labor that governs human life, *gives rise to* the "output" we observe—namely, masses of mass-disciplined, alienated "subjects" of labor.

This document constitutes an attempt to present in mathematical style the foundational elements of Marx's economic model as it is presented in all four volumes of *Capital*—in such state as we have them—*without introducing interpretative material from other authors*, for the limited purpose of answering the following question.

We wish to know, as precisely as possible, what Marx means when he describes the general rate of profit as *"the form in which capital becomes conscious of itself as social power"*:

> 'In capitalist production it is [...] a matter of extracting [...] /// for the capital advanced in production the same surplus-value or profit as any other capital of the same size, or a profit proportionate to its size, no matter in what branch of production it may be applied. The problem therefore is to sell commodities, and this is a minimum requirement, at prices which deliver the average profit [...] This is the form in which capital becomes conscious of itself as a social power, in which every capitalist participates in proportion to his share in the total social capital.' (C3, 296-297)

It is exactly this subject Marx is discussing when he delivers the following oft-quoted line:

> 'We thus have a mathematically exact demonstration of why the capitalists, no matter how little love is lost among them in their mutual competition, are nevertheless united by a real freemasonry vis-à-vis the working class as a whole.' (C3, 300)

We will begin, as *Capital* does, with the transformation of use-values into commodities (§2.1). We then present the asymmetric value-relation "IS WORTH" $\succ$ (Definition 2.4) and the symmetric value-relation "IS EQUAL IN VALUE TO" $\cong$ (Definition 2.8); these value-relations encode certain quantifiable relational features of the social objectification of labor (§2.3). The value-form is presented in §2.5.1; then the purchase of *labor-power* (Definition 2.6) can be discussed. In order to advance to the theory of the general rate of profit as quickly as possible, we introduce the components of constant capital, variable capital, and surplus-value in the context of the periodic circuit of commodity capital (§4.1; cf. C2, Ch. 20). We then present a working definition of capital (§5.1) and move on to Marx's theory of the general rate of profit (§5.3) as laid out in C3, Part 2.

Our intention is to represent the ideas in the text as faithfully as possible. We refer only to sources authored by Marx. Any divergence from the text that rises above the level of a technical detail should be attributed to the author's error. We believe that the novelty in this paper, if there is any,[5] consists solely of technical details related

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[3] In Deleuze and Guattari's work, the explanatory principles of Marx and Foucault come to speak with one voice—but the margins are too narrow to provide the proof here.

[4] Foucault, *Abnormal: Lectures at the Collège de France, 1974-1975*, passim.

[5] For example, countless authors and readers adopt algebraic vector notation, as we do, when summarizing Marx's mathematical models. A celebrated example is the Soviet-American economist Wassily Leontief, whose linear-algebraic generalization of the reproduction schema of C2, Part 3 appears in standard undergrad math textbooks, e.g. Lay, Lay and McDonald, *Linear Algebra and its Applications*, 6th ed. (2021), §2.6.

to bookkeeping and notation, e.g. our definition of *commodity-mass*.

2   The money-form and the objectification of human labor

> "I have popularised the passages concerning the substance of value and the magnitude of value as much as possible. The value-form, whose fully developed shape is the money-form, is very simple and slight in content. Nevertheless, the human mind has sought in vain for more than 2,000 years to get to the bottom of it." (Marx, "Preface to the First Edition," C1,[6] 89-90)

> "The second sheet especially bears rather strong marks of your carbuncles, but that cannot be altered now and I do not think you should do anything more about it in an addendum, for, after all, the philistine is not accustomed to this sort of abstract thought and certainly will not cudgel his brains for the sake of the form of value." (Engels, *Marx and Engels Collected Works*, 1987, vol. 42, 381)

In this section we present the theory of the value-form (*Wertform*) using modern mathematical language and symbolism. Some familiarity with set-theoretical binary relations will be advantageous, but is not required. A minimal degree of understanding of vector notation (e.g. $\mathbf{a} + \mathbf{a} = 2\mathbf{a}$) is recommended.

Two types of value-relation can be distinguished in Marx's theory of the value-form as he presents it in C1, Ch. 1 and in an appendix to the first edition[7] of C1. In order to distinguish between the two types of value-relation, it is first necessary to distinguish the roles played by the *relative form* and the *equivalent form* in Marx's theory, which in turn requires the review of the concepts of *use-value*, *commodity*, *exchangeability* and *the objectification of labor*. Once these preliminaries have been established, an asymmetric value-relation

$$\underbrace{\mathbf{X}}_{\text{RELATIVE FORM}} \quad \succ \quad \underbrace{\mathbf{Y}}_{\text{EQUIVALENT FORM}}$$
$$\underset{\substack{\text{IS WORTH} \\ \textit{(ist wert)}}}{}$$

and a symmetric value-relation

$$\mathbf{X} \quad \cong \quad \mathbf{Y}$$
$$\underset{\text{IS EQUAL IN VALUE TO}}{}$$

will be formally defined as set-theoretic binary relations.

Complaints of redundancy and obscurity are nearly universal among readers of the earliest pages of Volume 1, but we believe these readers' frustration is due in large part to the inadequacy of the mathematical notation available to Marx. The conflation *in symbols* of the relations "IS WORTH" and "IS EQUAL IN VALUE TO" with the everyday relation of numerical equality ($=$) renders Chapter 1 quite difficult to understand, as it collapses the successive stages of development of the value-form into a muddle of statements that seem—but indeed *only* seem—all to be saying more or less the same thing (e.g. in the various exchanges of linen for a coat). Yet it is evident that the text distinguishes between two senses in which a pair of commodities can confront each other. Our novelty in denoting the two types of value-relation by the symbols $\succ$ and $\cong$ is purely typographical.

We begin, as C1 does, with the distinction between *useful items* (a.k.a. *use-values*) and *commodities*.

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[6] Page numbers for C1 refer to the Penguin edition (1982) of *Capital*, Vol. 1, tr. B. Fowkes.
[7] Marx, "The Value-Form," Appendix to the 1st German edition of C1.

2.1 Use-values and commodities

The definitions of this subsection serve merely as means of bookkeeping. They are not intended to introduce any theoretical innovations.

Definition 2.1. *Capital* begins by defining a **use-value** [*Gebrauchswert*] to be a thing or substance[8] that has useful properties[9] which we will call its **uses**. A use-value is characterized and determined by its *quality* and *quantity*.[10] The following bulleted mathematical definitions provide a formal means of recording the quality and quantity of various articles (or "bodies," or *masses*) of use-value.

- We say that use-values are **equal in quality**, or that they are **of the same kind**, if there exists some set of uses they all possess. (In other words, we say that use-values $\mathbf{X}_1, \mathbf{X}_2, \ldots$ having respective sets $U_1, U_2, \ldots$ of useful properties are *equal in quality*, or *of the same kind*, if there exists some set V of useful properties such that each property in V is also in each of $U_1, U_2, \ldots$.)

- By a **mass** we mean a *continuous* or *discrete* positive quantity. A mass is **continuous** if its magnitude is measurable by an arbitrarily subdivisible unit (e.g. 1 lb.). By a **discrete** mass or positive quantity, we mean a nonempty finite set.

- A **continuous use-value mass** is a continuous mass of a substance that has useful properties. A **discrete use-value mass** is a discrete mass of use-values of the same kind. (For consistency with the text, we will simply write "a use-value" when we mean "a continuous or discrete use-value mass.")

Over time, humanity discovers new purposes for use-values, and also develops various standards of measurement for the mass of a use-value of this or that kind. Marx calls this the "work of history."[11] We therefore stipulate that:

- the uses of a thing or substance are a function of time and place, and

- the units used to measure a given kind of use-value depend on time and place.

By "a given time and place" we mean *a definite historical moment*, however the latter is defined.

Conventions:

- We will write the combined unit and "kind" of a use-value as an algebraic vector, e.g. $\mathbf{b} = 1$ lb. butter (so $2\mathbf{b} = 2$ lbs. butter).

- For discrete use-values, the unit will be omitted, e.g. $\mathbf{d} = 1$ Barbie Dreamhouse $= 1$ pc. Barbie Dreamhouse, where "pc." is the "piece" listed on invoices of manufacturers and retailers of such goods.

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[8] "The form of use-value is the form of the commodity's body itself, iron, linen, etc., its tangible, sensible form of existence. This is the natural form [*Naturalform*] of the commodity. As opposed to this the value-form (*Wertform*) of the commodity is its social form." ("The Value-Form")

[9] "The usefulness [translated as *utility* by Moore and Aveling] of a thing makes it a use-value." (C1, 126)

[10] "Every useful thing, for example iron, paper, etc., may be looked at from the two points of view of quality and quantity. [...] It is [...] the physical body of the commodity itself, for instance iron, corn, a diamond, which is the use-value or useful thing. When treating of use value, we always assume to be dealing with definite quantities, such as dozens of watches, yards of linen, or tons of iron." (C1, 125-126)

[11] "The discovery of these ways and hence of the manifold uses of things is the work of history. So also is the invention of socially recognized standards of measurement for the quantities of these useful objects." (C1, 125)

- We will use a capital boldface letter, e.g. $\mathbf{X}$, to represent a use-value in general, i.e. an arbitrary use-value of arbitrary kind and mass.

- We will use a lowercase boldface letter to represent the unit and kind of a use-value, and an italicized letter to represent the mass, e.g. $\mathbf{X} = x\mathbf{i}$.

Definition 2.2. A use-value (or use-value mass) is a **commodity** if it can be fairly exchanged for a use-value (mass) of a different kind[12] in some determinate proportion. We use the terms *commodity* and **commodity-mass** interchangeably.

In so far as they are mutually exchangeable, all commodities exist in a single, functionally universal context. We will call the universal set consisting of all commodities which can be exchanged at a given time and place the *market*.

Definition 2.3. Let $\mathfrak{M}$ denote the set of all commodity-masses that can be exchanged at a fixed time and place. We will call $\mathfrak{M}$ a (**commodity**) **market**. We also define

$$\mathcal{K}(\mathfrak{M}) = \left\{\mathbf{i} \,\middle|\, x\mathbf{i} \in \mathfrak{M} \text{ for some } x > 0\right\}$$

to be the set of kinds of commodity that appear in $\mathfrak{M}$ and

$$E(\mathbf{i}) = \left\{\mathbf{j} \in \mathcal{K}(\mathfrak{M}) \,\middle|\, \mathbf{j} \neq \mathbf{i}\right\} \qquad (\mathbf{i} \in \mathcal{K}(\mathfrak{M}))$$

to be the set of kinds of commodities appearing in $\mathfrak{M}$ other than a given kind $\mathbf{i} \in \mathcal{K}(\mathfrak{M})$.

Historical circumstances determine what kinds of commodity appear in the market. For instance, Mesoamerican maize presumably could not be traded in West Africa for Chinese gunpowder in the 11th century, since it is unlikely that both were ever found in the same market at that time and place.

2.2 Exchangeability and elementary forms of value

The *exchange-value*[13] [*Tauschwert*] of a commodity-mass is what remains of its value when use-value has been conceptually set aside, abstracted or "modded" out,[14] etc.

What remains of the commodity itself is what Marx calls the commodity's "social form," as opposed to its "natural form":[15] its *capacity for being fairly exchanged*, as opposed to its uses and physical attributes.[16]

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[12] "The product of labour is an object of utility in all states of society; but it is only a historically specific epoch of development which presents the labour expended in the production of a useful /// article as an 'objective' property of that article, i.e. as its value. It is only then that the product of labour becomes transformed into a commodity." (C1, 153-154)

[13] "Exchange-value appears first of all as the quantitative relation, the proportion, in which use-values of one kind exchange for use-values of another kind." (C1, 126)

[14] Roughly speaking, **modulo** means, "letting the following differences *vanish*"; in casual conversation among mathematicians, "**to mod out**" means to regard as equal to 0 (e.g. the kernel of a homomorphism). A simple example is that two whole numbers which differ by a multiple of 10, e.g. 23 and 93, are said to be equal *modulo* multiples of 10. Similar triangles provide a purely geometric example: they are equal in shape, modulo scale.

[15] "The form of use-value is the form of the commodity's body itself, iron, linen, etc., its tangible, sensible form of existence. This is the natural form (*Naturalform*) of the commodity. As opposed to this the value-form (*Wertform*) of the commodity is its social form." ("The Value-Form")

[16] "No scientist to date has yet discovered what natural qualities make definite proportions of snuff tobacco and paintings 'equivalents' for one another." (MECW 32, 317)

This capacity, or "**exchangeability**," consists in a commodity-mass's ability to be *posited*[17] *as equal in value to* a commodity-mass of a different kind.[18] Two different kinds of commodity $\mathbf{i}$ and $\mathbf{j}$ are equal in value if and only if they can be fairly exchanged in some proportion, x of $\mathbf{i}$ for y of $\mathbf{j}$. Thus to posit equality in value is to specify some proportion of exchange x/y.

The thing that actually does the positing is social convention. Social convention endows use-values with the property of exchangeability, conceptually transforming them[19] into commodities and establishing definite proportions of exchange.

For example, suppose a linen-producer and a coat-producer *in isolation* are haggling over the proportion in which their goods can be fairly exchanged.[20] Setting aside the physical properties of their respective commodities, their goods can be exchanged if and only if the two individuals reach an agreement as to how much of one is worth (*ist wert*) however much of the other. That is, the two individuals must agree to posit that, say, 20 yards of linen is equal in value to 1 coat. In this context, that of isolated barter, "social convention" simply means a consensus reached by two individuals.
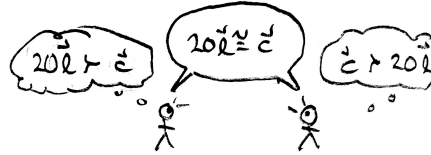

Figure 1: The linen-producer and the coat-producer coming to terms. (See footnote 20.)

When on the other hand exchangeability is collectively determined *by an entire society,* the phrase "social convention" refers to the "social interconnection" of "private labors," that is, a division of labor[21] that reflects the proportions in which commodities of different kinds exchange. Once we have defined *human labor*, *labor-power*, and the *value-form*, we will see that a determinate division of labor is encoded in the value-form as a sequence of fractions we call *proportions of exchange*.

Identifying the property of "having an exchange-value" with "exchangeability," and thus with a definite proportion of exchange, x of this for y of that, we define "exchangeability" *relationally*: the most elementary statement of the exchangeability of a commodity-mass $x\mathbf{i}$ is a statement of the form

$$x\mathbf{i} \text{ IS WORTH } y\mathbf{j},$$

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[17] The *ability* to be exchanged (for something) is the *ability* to be equal in value (with that something). To endow a use-value with the property of exchangeability therefore means to affirm, or *posit*, its ability to be equal in value to some other use-value. The idea that exchanging involves *positing exchange-value* appears in the *Grundrisse* (written 1857-1858, quotes to follow). Note however that the main text of *Capital* does not emphasize this aspect of exchange. ◇◇◇ "[T]he act of exchange [...] constitutes the positing as well as the proving of the exchange values and of the subjects as exchangers" ◇◇◇ "The exchange of the overflow is a traffic which posits exchange and exchange value." ◇◇◇ "The individual can employ money only by divesting himself of it, by positing it as *being for others,* in its social function." ◇◇◇ "what is circulated by money is exchange value, hence *prices*. [...] Circulation is the positing of prices, it is the process in which commodities are transformed into prices: their realization as prices." (*Grundrisse*, 242, 257, 187, 228n.)

[18] "The product of private labor [...] *only* has *social form* insofar as it has *value-form* and hence the *form of exchangeability* with other products of labor." ("The Value-Form")

[19] See footnote 12.

[20] "Let us consider exchange between linen-producer A and coat-producer B. Before they come to terms, A says: 20 yards of linen are worth 2 coats (20 yards of linen $= 2$ coats), But B responds: 1 coat is worth 22 yards of linen (1 coat $= 22$ yards of linen). Finally, after they have haggled for a long time they agree: A says: 20 yards of linen are worth 1 coat, and B says: 1 coat is worth 20 yards of linen." ("The Value-Form")

[21] "The social interconnection of these private labors exists materially, insofar as they are members of a naturally evolved social division of labor and hence, through their products, satisfy wants of different kinds, in the totality of which the similarly naturally evolved system of social wants consists." ("The Value-Form")

that is,

"the value of x units of $\mathbf{i}$ is expressed in y units of $\mathbf{j}$,"

where $\mathbf{j}$ is a kind of commodity other than $\mathbf{i}$.[22] We will call such a statement an *elementary form of value*, and write it in symbolic shorthand as $x\mathbf{i} \succ y\mathbf{j}$; a formal definition follows.

Definition 2.4. For any two commodity-masses $x\mathbf{i}$ and $y\mathbf{j}$ which are not equal in quality (that is, $\mathbf{i} \neq \mathbf{j}$), we write $x\mathbf{i} \succ y\mathbf{j}$, and we say $x\mathbf{i}$ **is worth** (*ist wert*) or **exchanges for** $y\mathbf{j}$, if and only if $x\mathbf{i}$ (the **relative form**) expresses its value in $y\mathbf{j}$ (the **equivalent form**).

$$\underbrace{x\mathbf{i}}_{\text{RELATIVE FORM}} \quad \underset{\substack{\text{IS WORTH} \\ \text{(ist wert)}}}{\succ} \quad \underbrace{y\mathbf{j}}_{\text{EQUIVALENT FORM}}$$

A single statement of the form $x\mathbf{i} \succ y\mathbf{j}$ for a particular pair of commodities $x\mathbf{i}$ and $y\mathbf{j}$ such that $\mathbf{i} \neq \mathbf{j}$ is called an **elementary** (or **simple**) **form of value**.[23]

Note that "the exchange-value of a commodity-mass $x\mathbf{i}$" is undefined in the absence of an elementary form of value relating $x\mathbf{i}$ to some $y\mathbf{j}$ ($y > 0$, $\mathbf{j} \neq \mathbf{i}$).

2.3 The objectification of labor and equality in value

In certain modes of production, according to Marx, buyers and sellers come to agree about what constitutes a "*fair* exchange" based on the amount of labor required to produce this or that commodity. The amount of labor required for a commodity's production is then called its **value**,[24] and two commodities are said to be *equal in value* if the same amount of labor is expended in the production of each.

But before buyers and sellers can quantitatively compare labor of different kinds, e.g. weaving and tailoring, some common unit for labor of *all* kinds must first have been established. We will call the establishment of such a common unit the *objectification of labor*.

In this section we summarize Marx's analysis of the objectification of labor in C1, and then formally define *equality in value*.

Definition 2.5. By **social labor** (or **socially necessary labor**, or **socially necessary labor-time**) we mean the amount of labor-time needed to produce a given commodity mass "under the conditions of production normal for a given society and with the average degree of skill and intensity of labor prevalent in that society." (C1, 129) The **total social labor** is the total socially necessary labor required to produce the social wealth.[25]

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[22] "The simplest relation of the commodity is obviously *its relation to a single other commodity*, no matter *which one*. Hence *the relation of two commodities furnishes the simplest value-expression for a commodity*." ("The Value-Form")

[23] "A commodity's simple form of value is contained in its value-relation with another commodity of a different kind, i.e. in its exchange relation with the latter. The value of commodity A is qualitatively expressed by the direct exchangeability of commodity B with commodity A. It is quantitatively expressed by the exchangeability of a specific quantity of commodity B with a given quantity of A." (C1, 152) Moore and Aveling translate the first sentence quoted as: "The elementary form of value of a commodity is contained in the equation, expressing its value relation to another commodity of a different kind, or in its exchange relation to the same."

[24] See footnote 12.

[25] By **social wealth** we mean the a society's total material means of satisfying human wants, that is, the sum total of useful items available for consumption.

- Note that differences among the workers' level of skill and the factories' level of technology are immaterial to the determination of value = socially necessary labor.

- If a commodity can be produced under average conditions by a worker of average skill in *less* time, its value *decreases* (C1, 130)—in Marxian economics, this is a tautology.

$$\underset{\text{labor productivity}}{\text{average conditions of}} \xrightarrow{\text{determine}} \underset{\substack{\text{socially necessary} \\ \text{labor-time}}}{\text{amount of}} \overset{\text{is identical with}}{\equiv} \underset{\text{value}}{\text{magnitude of}}$$
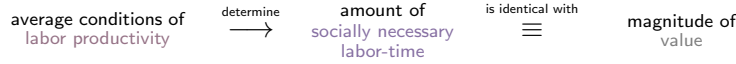
Labor-power simply means the ability to do labor. We will use the phrases "labor-power expended," "labor-time," and "labor" interchangeably from this point forward.

Definition 2.6. A human being's **labor-power** is their capacity[26] to produce use-values by their labor,[27] assumed (for simplicity) to be identical for all individuals,[28] conceived in theory as a fraction of the total social labor-power,[29] and measured in practice in units of time the individual is able and available[30] to work. The expenditure of labor-power is *labor*, measured in time.

Axiom 1 (*Objectification of labor*). One hundred percent of the total social labor (= labor-power expended) is identical with one hundred percent of the value it creates.[31]

$$\text{OBJECTIFICATION OF LABOR:} \qquad \underset{\substack{\text{by society} \\ \textit{(units of labor-time)}}}{\underset{\text{labor-power expended}}{\text{total socially necessary}}} = \underset{\substack{\text{by society} \\ \textit{(units of labor-time)}}}{\underset{\text{commodities produced}}{\text{total value of}}}$$

Definition 2.7. We will call the labor-power that must be expended in the production of a commodity-mass $x\mathbf{i}$ the **value of $x\mathbf{i}$ as a function of mass**, and we denote it by $L_{\mathbf{i}}(x)$; it is measured in time.

We assume the functions $L_{\mathbf{i}}$ are linear:

Axiom 2 (*Value is proportional to mass*). For any commodity-mass $x\mathbf{i}$, the labor-power $L_{\mathbf{i}}(x)$ that must be expended in its production is proportional to (i.e. is a linear function of) its mass, x.

- Clearly if we denote $\rho_{\mathbf{i}} = \dfrac{x}{L_{\mathbf{i}}(x)}$ we can write $L_{\mathbf{i}}(x) = \dfrac{1}{\rho_{\mathbf{i}}} x$ (units/hr) or even $L_{\mathbf{i}} = \rho_{\mathbf{i}}^{-1}$ (hr/unit).

- Thus $L_{\mathbf{i}}(x) = x/\rho_{\mathbf{i}} = xL_{\mathbf{i}}(1)$ for any suitable mass $x > 0$.

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[26] "We mean by labor-power, or by labor-capacity, the aggregate of those mental and physical capabilities existing in the physical form, the living personality, of a human being, capabilities which he sets in motion whenever he produces a use-value of any kind." (C1, 270)

[27] "Tailoring and weaving, although they are qualitatively different productive activities, are both a productive expenditure of human brains, muscles, nerves, hands etc., and in this sense both human labor. They are but two different forms [*modes* in Moore and Aveling] of expending human labor-power." (C1, 134)

[28] "**simple labor-power** [. . . is] the labor-power possessed in his bodily organism by every ordinary man, on the average, without being developed in any special way. [. . .] In the interests of simplification, we shall henceforth view every form of labor-power directly as simple labor-power;" (C1, 135)

[29] "The total labor-power of society, which is manifested in the values of the world of commodities, counts here as one homogeneous mass of human labor-power, although composed of innumerable individual units of labor-power." (C1, 129)

[30] "In order that [the] possessor [of labor-power] may sell it as a commodity, he must have it at his disposal, he must be the free proprietor of his own labor-capacity, hence of his person. [. . .] the proprietor of labor-power must always sell it for a limited period only, for if he were to sell it in a lump, once and for all, he would be selling himself, converting himself from a free man to a slave, from an owner of a commodity into a commodity." (C1, 271)

[31] "the labor that forms the substance of value is equal human labor, the expenditure of identical human labor-power." (C1, 129)

Definition 2.8. For any two commodity-masses $x\mathbf{i}$ and $y\mathbf{j}$ such that $\mathbf{i} \neq \mathbf{j}$, we write $x\mathbf{i} \cong y\mathbf{j}$, and we say $x\mathbf{i}$ and $y\mathbf{j}$ are **equal in value**, if and only if the labor-power that must be expended in order to produce x units of $\mathbf{i}$ is equal to the amount of labor-power that must be expended in order to produce y units of $\mathbf{j}$. That is,

$$x\mathbf{i} \cong y\mathbf{j} \iff L_{\mathbf{i}}(x) = L_{\mathbf{j}}(y) \qquad (\mathbf{i} \neq \mathbf{j}).$$

Axiom 3 (*Commodities exchange at their value*)**.** If $x\mathbf{i} \succ y\mathbf{i}$, then $x\mathbf{i} \cong y\mathbf{i}$. In words: for x units of $\mathbf{i}$ to be worth y units of $\mathbf{j}$, i.e. for the value of $x\mathbf{i}$ to be expressed in $y\mathbf{j}$, it is a necessary condition that the amount of labor-power that must be expended in order to produce $x\mathbf{i}$ is equal to the amount of labor-power that must be expended in order to produce $y\mathbf{j}$.

In broad terms, the difference between the relations $\succ$ and $\cong$ is that $\succ$ is asymmetric, as evidenced by Marx's distinction between the relative and equivalent forms, while $\cong$ is symmetric due to the symmetry of the relation of numerical equality $L_{\mathbf{i}}(x) = L_{\mathbf{j}}(y)$. In the next section, we explore the differences between the relative and equivalent forms.

2.4   The relative and equivalent forms

The relative form $x\mathbf{i}$ and the equivalent form $y\mathbf{j}$ in an elementary form of value $x\mathbf{i} \succ y\mathbf{j}$ are related as "poles."[32] According to "The Value-Form," this means:

 (i) It is meaningless to speak of a relative form in the absence of a related equivalent form, and vice versa.[33]

 (ii) The relative and equivalent forms are always unequal in kind.[34]

 (iii) A single value is expressed in an elementary form, hence is common to both the relative and equivalent forms.[35]

These properties are well expressed in terms of the structure of a set-theoretic **binary relation on a set**, i.e. a set of ordered pairs $(\mathbf{X}, \mathbf{Y})$, where the terms $\mathbf{X}$ and $\mathbf{Y}$ are required to be members of some fixed choice of set. Given a commodity market $\mathfrak{M} =$ (set of commodities that can be exchanged at a given time and place), regard all elementary forms of value $\mathbf{X} \succ \mathbf{Y}$ as ordered pairs $(\mathbf{X}, \mathbf{Y})$. Then the set (which we denote by the symbol $\succ$) of all such ordered pairs is a binary relation on $\mathfrak{M}$. We can similarly understand $\cong$ as a binary relation on $\mathfrak{M}$. We will call $\succ$ and $\cong$ (**asymmetric** and **symmetric**) **value-relations** (respectively).

Figure 2 illustrates how in our schema the value-relations $\succ$ and $\cong$ vary depending on (the membership of) $\mathfrak{M}$, which in turn varies depending on time and place.

Now:

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[32] "The Value-Form," I.§1, "The two poles of the expression of value (*Wertausdruck*): relative form and equivalent form," passim.

[33] "Relative value-form and equivalent form are moments of the *same expression of value,* which belong to one another and are reciprocally conditioning and inseparable." ("The Value-Form")

[34] "[The relative and equivalent] forms are mutually excluding or opposed extremes, i.e. *poles*, of the same expression of value. They are always *distributed* amongst *different* commodities, which the expression of value relates to one another." ("The Value-Form")

[35] "The coat is only the *same* as the linen to the extent that both are *values*. Thus that the linen is related to the coat as to *something of its own kind* or that the coat *as a thing of the the same substance is equated* to linen, expresses the fact that *the coat counts in this relation as value*. It is *equated* to the linen insofar as the latter *is value* as well." ("The Value-Form")

Figure 2: Schematic of the formal dependence of the market $\mathfrak{M}$
and the relations $\succ$ and $\cong$ on historical conditions.
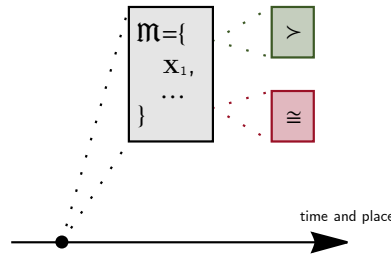
- Property (i) is enforced by requiring that no term in a pair which is a member of the binary relation $\succ$ can appear as a member of $\succ$ without its partner. Since the members of $\succ$ are by definitions pairs of commodity-masses, this property is satisfied.

- Property (ii) is equivalent to the requirement that if $x\mathbf{i} \succ y\mathbf{j}$, then $\mathbf{i} \neq \mathbf{j}$. This is ensured by the definition of an elementary form of value $x\mathbf{i} \succ y\mathbf{j}$.

- Property (iii) can be understood as saying that a statement of the form $x\mathbf{i} \succ y\mathbf{j}$ cannot be decomposed into *unary* statements of value, by which we mean statements which refer to a single term without reference to any other. An *equivalence* is asserted in an elementary form of value $x\mathbf{i} \succ y\mathbf{j}$, and no further simplification of the latter expression is possible if the equivalence is to be affirmed. The relation $\succ$ satisfies Property (iii) by definition of a binary relation as a set of *pairs*, provided that we interpret each pair $x\mathbf{i} \succ y\mathbf{j}$ as denoting a single equivalence of value.

The relative form is characterized in "The Value-Form" as follows:

(iv) The value of the relative form consists of how much of the equivalent it is worth. Put another way, the relative form "expresses its value in the body of"[36] a commodity of a different kind,[37] called the equivalent form.

(v) The "initiative proceeds from" the relative form.[38]

(vi) The relative form counts "as the shape" [*als Gestalt*] of use-value.[39]

In the context of barter between two isolated individuals, Property (iv) is self-explanatory. Exactly two elementary forms of value determine the transaction, one the reverse of the other. Both parties must posit that the two commodity-masses $x\mathbf{i}$ and $y\mathbf{j}$ to be exchanged are equal in value, $x\mathbf{i} \cong y\mathbf{j}$, and in this context, the two masses $x\mathbf{i}$ and $y\mathbf{j}$ can be posited as equal in value only if $x\mathbf{i}$ is worth $y\mathbf{j}$ to one party and $y\mathbf{j}$ is worth $x\mathbf{i}$ for the other

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[36] "[The relative form] *expresses its value in the body of a commodity different from it* [...] we say of *the commodity which expresses its value in another commodity*: its value is represented as *relative value*, or is in the *relative value-form*." ("The Value-Form")

[37] "the value of the commodity is expressed as something completely different from its own sensible existence" ("The Value-Form")

[38] "Since it is the linen *which is to express its value,* the initiative proceeds from it. It *enters into a relation* with the coat, i.e. *with some other commodity different from itself.* This relation is a relation of equalization (*Gleichsetzung*)." ("The Value-Form")

[39] "In the *relation of value* of the linen to the coat the natural form (*Naturalform*) of the linen counts only *as the shape* (*als Gestalt*) *of use-value,* the natural form of the coat only as *value-form* (*Wertform*) or *shape* (*Gestalt*) of exchange-value. The *inner opposition* between use-value and value (*Gebrauchswert und Wert*) contained in a commodity is thus *represented* by an *external opposition*, i.e. *the relation of two commodities,* of which the one counts immediately only as use-value, the other immediately only as exchange-value, or in which the two opposing determinations, use-value and exchange-value, are distributed *in a polar* manner among the commodities. If I say: *As a commodity* the linen is use-value and exchange-value, this is my judgement about the *nature* of the commodity gained by analysis. As opposed to this, in the expression '20 *yards of linen* = 1 *coat*' or '20 *yards of linen are worth* 1 *coat*' the linen itself says that it 1. is a *use-value* (linen); 2. is an *exchange-value* distinct from that (something equal to the coat); and 3. is the *unity of these two differences,* and thus is a *commodity.* " ("The Value-Form")

(Figure 1). In barter there is perfect democracy, even if it is only a democracy of two, and even if exigencies compel one or both parties to make the exchange under duress.

Property (v) appears somewhat more mysterious. We make the following interpretation:

The asymmetry of an elementary form of value $\mathbf{X} \succ \mathbf{Y}$ ("$\mathbf{X}$ is worth $\mathbf{Y}$") expresses the fact that the exchange of $\mathbf{X}$, the relative form, is *necessary*.

What is produced and immediately consumed by the producer—say, compost destined for use in one's own garden—never confronts other use-values as a commodity *simply because it doesn't have to.* The utility of the compost can be realized independent of any exchange. In this situation the compost functions not as an exchangeable commodity, but only as a use-value.

To the extent that the value of what one produces *cannot* be realized except through exchange, on the other hand, its value consists of what it can be exchanged for. In other words, its value consists of "value, setting aside use-value"—that is, *exchange-value*.

Properties (v) and (vi) can now be understood as saying (v) that the *motivation* to exchange lies with the possessor of the relative form and (vi) that the use-value of the relative form is *abstracted out* in the exchange, so that the relative form counts only as the "*shape*" of use-value, disregarding its actual uses.

Examples:

- In the context of barter, only human beings and use-values are at stake. Suppose I have a surplus of corn which is of no use to me, and I need linen. I realize the relative value of my corn by exchanging it for the use-value I need, linen. The (relative) value of my surplus corn is expressed in (the equivalent value of) the linen I can get for it.

- In a society where wages are the predominant means of obtaining the means of subsistence, labor-power, the ability to work, must occupy the position of the relative form, since that is what the workers must exchange. In so far as the workers do not possess the means of production, they have only their labor-power to dispose of in trade. They are therefore compelled to realize the value of their ability to work by exchanging it, either directly for means of subsistence (e.g. payment in kind) or for some intermediating means of exchange (money, say) which can in turn be traded for means of subsistence.

- In terms of different **trades**, by which we mean branches of production that respectively produce different kinds of commodity, the goods produced in each trade must be exchanged in order for their value to be realized. The trades may collectively barter, i.e. exchange in kind, but in a society that measures value in money, all the trades must exchange some of their product for cash. Here the equivalent form is money and

the relative form is the variable

$$\mathbf{X}_i = x_i \mathbf{a}_i$$

$$= \text{total commodity-mass produced by trade } i$$

$$= \text{units of commodity } \mathbf{a}_i \text{ produced by trade } i.$$

The exchange of commodities implies a difference in uses, hence a difference in the type of labor that produced them.[40] A complex system of commodity-flows subsequently develops between the various trades that carry out the various types of labor required by the society. This system is the material expression of the general social *division of labor* in a society governed by commodity exchange.[41] In this sense *a set of elementary forms of value encodes a quantifiable division of labor in the material world*.

The equivalent form is characterized in "The Value-Form" as follows:

(vii) The equivalent form counts as value for a commodity of a different kind.[42]

(viii) The equivalent form has an immediately social form.[43]

 (ix) The equivalent form "has the social quality of being immediately exchangeable".[44] It "counts as the visible incarnation" of value, and does not need to be exchanged in order to count as value and to act as value.

  (x) The equivalent form counts as the "shape" of exchange-value.

If the value of the relative form consisted in its uses, there would be no need for the relative form to express its value in a commodity of some other kind, as Property (vii) requires. Therefore, for the equivalent form to "count as" the value of the relative form, the uses of the relative form must be conceptually set aside. The value of the relative form can then be measured simply in units of mass of the equivalent form, e.g. 6 hours of labor-power is worth $3/20$ pounds sterling.

Property (viii) presumably implies that the relative form is *not* "immediately social." In so far as the relative form *is* social, then, it must be mediated in the equivalent form. "The social" here refers to the establishment of equality in value, so Property (viii) says that the value of the relative form is established only through its relation to the equivalent form, while *the value of the equivalent form does not need to be established through exchange*. To hoard endlessly may be practically impossible for the individual and detrimental to society as a whole, but

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[40] "If the use-values were not qualitatively different, hence not the products of qualitatively different forms of useful labor, they would be absolutely incapable of confronting each other as commodities. Coats cannot be exchanged for coats, one use-value cannot be exchanged for another of the same kind." (C1, 132)

[41] "In a society whose products generally assume the form of commodities, i.e. in a society of commodity producers, this qualitative difference between the useful forms of labor which are carried on independently and privately by individual producers develops into a complex system, a social division of labor." (C1, 133)

[42] "The linen is the commodity which expresses its value in the body of a commodity different from it, the coat. On the other hand, the commodity-type coat serves as the material in which value is expressed." ("The Value-Form")

[43] "The product of private labor [...] has *immediately social form* insofar as its own bodily or natural form is *at the same time* the form of its exchangeability with other commodities or *counts as value-form for another commodity*." ("The Value-Form")

[44] "Hence, for example, the riddle of *gold*, that seems to possess, by nature, apart from its other natural properties, its colour, its specific weight, its non-oxydisability in air, etc., also the equivalent-form, or the social quality of being *immediately exchangeable* with all other commodities." ("The Value-Form")

it cannot typically be said that a hoard becomes worth less simply because it has grown too large, holding all other economic variables approximately constant. The worth of the Crown's gold reserves does not depend on its being actually exchanged for any other kind of commodity. The exchange of the equivalent form is in general not *motivated* by a need to establish value, unlike the relative form (Property v), whose value could not be measured by the amount on hand in isolation, but could only be reckoned in terms of the mass of an equivalent for which the amount on hand exchanges. As an important special case, the equivalent form can be used to purchase a commodity without having to swap it for some intermediary third commodity (Property ix); that is, it acts as value (e.g. here, as means of purchase).

When *money in the form of gold* comes to play the role of a (universal) equivalent, we must deny that any amount of gold $y\mathbf{g}$ IS WORTH any amount of another kind of commodity. In the context of a single elementary form of value $x\mathbf{i} \succ y\mathbf{g}$ the equivalent form $y\mathbf{g}$ cannot be said to BE WORTH *anything*. Here we are stating nothing more or less than a grammatical fact: in the statement "$x\mathbf{i}$ IS WORTH $y\mathbf{g}$," the equivalent form $y\mathbf{g}$ is the object and not the subject of the verb.

Our *interpretation* of what is grammatically obvious is that *the worth of the equivalent form is a social fact.* The association of a particular kind of commodity (gold, say) with the equivalent form is a matter of social convention, which constitutes the end-result of complex and longstanding historical processes. In so far as this or that kind of commodity comes to be associated with the equivalent form, its value comes to be understood as *that in terms of which the value of something else is measured.* In this sense the equivalent form is the "shape" of exchange-value (Property x): it is insensitive to what it can be fairly exchanged for in so far as what it can be fairly exchanged for is not what it is worth.

The equivalent form is "immediately social": this means that the social convention is to attribute "worth" to it *directly.* Thus the society lays down the conceptual groundwork for commodity fetishism; if the worth of gold is attributed to gold directly, without the mediation of any other kind of commodity, then one is more likely to mistake its worth for an intrinsic, "natural" property.

2.5   Value-forms in general

A *value-form* considered in isolation, "in itself," is a set of quantitative relations between commodity-masses that specify how much of each kind of commodity can be fairly exchanged for another kind at a given time and place (i.e. in a given market $\mathfrak{M}$). The *money-form* [*Geldform*] is a particular type of value-form; that is, a money-form is a value-form that satisfies certain properties in addition to those properties shared by all value-forms.

Speaking less formally, a value-form is a projection of the universe of human wants into a parallel space of exchangeable items which are created by human labor and which exchange in proportions that are the result of social consensus. In this sense, the value-form encodes social relations; in particular, the proportions of exchange

associated with a value-form describe some definite division of labor.

At a further remove, the value-form encodes various different kinds of concrete human labor as "labor in general," or *abstract labor*: the differences between various types of labor are, so to speak, "zeroed out" in so far as commodities are considered only in terms of their exchange-value and not in terms of their provenance (that is, the tale of their production) or their utility (their ability to satisfy human wants).

The theory of the value-form thus facilitates the analysis and critique of those institutions and techniques by means of which the capitalist economic order brings about a generalized objectification of human labor and the useful items human labor produces, making it possible to speak of the moral imperative of "abolishing the value-form" or even "abolishing labor." These phrases do not express a desire for the abolition of human endeavor or of the will to create, but rather call for resistance to the systemic social transformation of human life, and in particular human ingenuity and all its fruits, into a homogeneous mass of dollar-values, distributed in the forms of money and consumable goods.

Although the value-form *at any fixed time and place* is (tautologically) static and unchanging, it is nonetheless impacted by real-world factors not directly represented in the quantitative relations $x\mathbf{i} \succ y\mathbf{j}$ it contains. If time and geographical location are not held fixed, then the proportions of exchange x/y tend to vary, new kinds of commodity tend to emerge, and old kinds tend to vanish. In this sense the old value-form passes away and a new one is born[45] with every variation in proportions of exchange, every introduction of a new kind of commodity to the market, and every disappearance of an existing kind.

2.5.1 Definition of a value-form and basic results

Definition 2.9. A **value-form for a market** $\mathfrak{M}$ is a set $\mathcal{V}$ of elementary forms of value such that

(1) each pair of different kinds of commodity $\mathbf{i}, \mathbf{j} \in \mathcal{K}(\mathfrak{M})$ ($\mathbf{i} \neq \mathbf{j}$) appearing in $\mathfrak{M}$ is represented by at most one elementary form of value in $\mathcal{V}$, that is, either $x\mathbf{i} \succ y\mathbf{j}$ or $y\mathbf{j} \succ x\mathbf{i}$ for some $x > 0$ and $y > 0$, and

(2) every kind of commodity $\mathbf{i} \in \mathcal{K}(\mathfrak{M})$ appearing in $\mathfrak{M}$ appears in at least one elementary form of value in $\mathcal{V}$.

Definition 2.10. Let $\mathcal{V}$ be a value-form for a market $\mathfrak{M}$. Let us call the fractions $x/y > 0$ for elementary forms of value $x\mathbf{i} \succ y\mathbf{j}$ in $\mathcal{V}$ **proportions of exchange**. By the **division of labor** encoded in $\mathcal{V}$, we mean the map $\mathcal{V} \mapsto (0, \infty)$ defined by assigning to each elementary form of value $x\mathbf{i} \succ y\mathbf{j}$ in $\mathcal{V}$ the corresponding proportion of exchange x/y.

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[45] That is, if a duration of time and/or a geographical region is specified, we imagine a sequence or continuum of value-forms, identifying each value-form with a specific time and place. In mathematicians' jargon, the structure we have in mind is exactly that of a bundle whose fibers F_b over $b \in B$ are (possibly non-isomorphic) value-forms and whose base space B represents time and/or place (e.g., for a fixed choice of location, identify B with time, e.g. $B = (-c, c)$, $c > 0$ for a definite interval of continuous time, $B = \{\ldots, -1, 0, 1, \ldots\}$ for an unending sequence of instants, etc.). Clearly the bundle has local product structure exactly where all fibers (value-forms) over an interval in B are isomorphic.

If we wish to visually represent a value-form $\mathcal{V}$ in human-readable form, we can list the elementary forms of value $\mathbf{X} \succ \mathbf{Y}$ in $\mathcal{V}$ in a table, e.g.:

RELATIVE	$\underset{\text{IS WORTH}}{\succ}$	EQUIVALENT
20 yd. linen		1 coat
20 yd. linen		1 qr. corn

Writing $\mathbf{a}_1 = 1$ yd. linen, $\mathbf{a}_2 = 1$ coat, $\mathbf{a}_3 = 1$ qr. corn, the same information can be written more compactly:

$$20\mathbf{a}_1 \succ \mathbf{a}_2$$

$$20\mathbf{a}_1 \succ \mathbf{a}_3$$

In this example,[46]

$$\mathcal{V} = \big\{ 20\mathbf{a}_1 \succ \mathbf{a}_2, \ 20\mathbf{a}_1 \succ \mathbf{a}_3 \big\}.$$

The following result facilitates calculations of value.

Proposition 1. If $x\mathbf{i} \cong y\mathbf{j}$, then $t\mathbf{i} \cong \dfrac{y}{x} t\mathbf{j}$ $(t > 0)$.

Proof. Suppose $x\mathbf{i} \succ y\mathbf{j}$ $(x > 0, y > 0)$, and let $t > 0$. Then $x\, L_\mathbf{i}(1) = y\, L_\mathbf{j}(1)$ since by Axiom 2 each of $L_\mathbf{i}$ and $L_\mathbf{j}$ is linear. Now $L_\mathbf{i}(t) = t\, L_\mathbf{i}(1) = \dfrac{ty}{x} L_\mathbf{j}(1) = L_\mathbf{j}\left(\dfrac{ty}{x}\right)$, which yields $t\mathbf{i} \cong \dfrac{ty}{x}\mathbf{j}$. $\qquad\square$

The next result establishes that $\cong$ is reflexive, symmetric, and transitive, from which it follows that the quotient $\mathfrak{M}/\cong$ can be formed.[47]

Proposition 2. Suppose $x\mathbf{i} \succ y\mathbf{i}$ and $y\mathbf{j} \succ z\mathbf{k}$. Then $x\mathbf{i} \cong x\mathbf{i}$, $y\mathbf{j} \cong x\mathbf{i}$, and $x\mathbf{i} \cong z\mathbf{k}$.

Proof. Obviously $L_\mathbf{i}(x) = L_\mathbf{i}(x)$, giving $x\mathbf{i} \cong x\mathbf{i}$. Since $L_\mathbf{i}(x) = L_\mathbf{j}(y) = L_\mathbf{k}(z)$ by Axiom 3, we have $y\mathbf{j} \cong x\mathbf{i}$ and $x\mathbf{i} \cong z\mathbf{k}$. $\qquad\square$
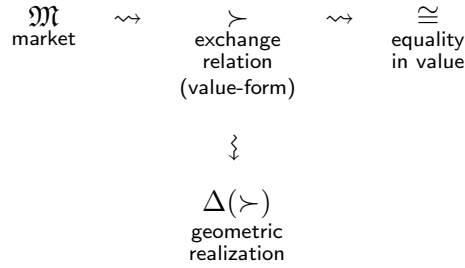
Definition 2.11. Let $\mathcal{V}$ be a value-form for a given market $\mathfrak{M}$. Write $\mathbf{i} \diamond \mathbf{j}$ for $\mathbf{i}, \mathbf{j} \in \mathcal{K}(\mathfrak{M})$ $(\mathbf{i} \neq \mathbf{j})$ if and only if there exist $x > 0$ and $y > 0$ such that $x\mathbf{i} \succ y\mathbf{j}$ or $y\mathbf{j} \succ x\mathbf{i}$ is in $\mathcal{V}$. A **path in $\mathcal{V}$ from i to j** is a sequence of statements $\mathbf{a}_1 \diamond \mathbf{a}_2$, $\mathbf{a}_2 \diamond \mathbf{a}_3$, ..., $\mathbf{a}_{k-1} \diamond \mathbf{a}_n$, where $n = 2, 3, 4, \ldots$, such that $\mathbf{a}_1 = \mathbf{i}$ and $\mathbf{a}_n = \mathbf{j}$.

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[46] Here the market $\mathfrak{M}$ can be defined either (1) as the set of commodity-*masses* actually appearing in a value-form $\mathcal{V}$, e.g. $\mathfrak{M} = \{20\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$, or (2) as the union of all *possible* commodity-masses, e.g. $\mathfrak{M} = \bigcup_{i=1}^{3} \{x_i\mathbf{a}_i \mid x_i > 0\}$. In either case, $\mathcal{V}$ represents the minimal information needed to represent a particular division of labor, e.g. $\{(\mathbf{a}_1, \mathbf{a}_2) \mapsto 20/1, (\mathbf{a}_1, \mathbf{a}_3) \mapsto 20/1\}$. If convention (1) is adopted, an elementary form of value for two commodity-masses $x\mathbf{i}, y\mathbf{j} \notin \mathfrak{M}$ such that $ux\mathbf{i} \succ uy\mathbf{j}$ for some $u > 0$ (in our example, say $40\mathbf{a}_1$ and $2\mathbf{a}_2$) can be extrapolated: $(20\mathbf{a}_1 \succ \mathbf{a}_2 \implies 40\mathbf{a}_1 \succ 2\mathbf{a}_2)$. If convention (2) is adopted, then in order to completely flesh out the formalization it would be necessary to specify the domain of each coefficient x_i for $x_i\mathbf{a}_i \in \mathfrak{M}$ more carefully than we have indicated, say, if any of the $x_i\mathbf{a}_i$ represent discrete masses, or commodities that are in limited supply.

[47] Recall that the elements of a quotient of a set by an equivalence relation defined on that set are equivalence classes. In this case, the equivalence classes $[x\mathbf{i}] = [x\mathbf{i}]_\cong = \{y\mathbf{j} \in \mathfrak{M} \mid y\mathbf{j} \cong x\mathbf{i}\}$ are sets of commodities which are equal in value. We might call these sets of commodities $[x\mathbf{i}]$ **exchange-forms**. They conceptually correspond to *distinguishable exchange-values*, e.g. $[\mathbf{i}] \neq [2\mathbf{i}]$. (To see that $[\mathbf{i}] \neq [2\mathbf{i}]$ for any kind of commodity $\mathbf{i}$, observe that $\mathbf{i} \cong 2\mathbf{i}$ implies $L_\mathbf{i}(1) = 0$, contradicting that $L_\mathbf{i}$ is linear.) Suppose now that a function $\varepsilon : \mathfrak{M}/\cong \to (0, \infty)$ is defined. It would be meaningless to ask, given only the *output* $p = \varepsilon([x\mathbf{i}])$ of such a function ε, to which particular representative (*use-value*)—$x\mathbf{i}$ or some other $y\mathbf{j} \in [x\mathbf{i}]$—the output p belongs. In other words, the quotient construction allows us to formalize the notion that, by being mediated through the relation of equality in value, exchange-value "*forgets*" use-value. Cf. footnotes 14 and 52.

Proposition 3. If there is a path from $\mathbf{i}$ to $\mathbf{j}$ in a value-form $\mathcal{V}$, then there exist $x > 0$ and $y > 0$ such that $L_{\mathbf{i}}(x) = L_{\mathbf{j}}(y)$.

We now give a geometric construction of a directed graph (diagram of arrows) that summarizes a given value-form. We use it to visualize a given value-relation $\succ$. Illustrations follow in the next subsection. Schematically, our process of constructing $\cong$ as a relation defined on $\mathfrak{M}$ is as follows:

$$
\begin{array}{ccccc}
\mathfrak{M} & \rightsquigarrow & \succ & \rightsquigarrow & \cong \\
\text{market} & & \text{exchange} & & \text{equality} \\
& & \text{relation} & & \text{in value} \\
& & \text{(value-form)} & &
\end{array}
$$

$$
\begin{array}{c}
\updownarrow \\
\Delta(\succ) \\
\text{geometric} \\
\text{realization}
\end{array}
$$

Definition 2.12. The **geometric realization** of a value-relation $\succ$ is a directed graph $\Delta(\succ)$ constructed as follows.

- The vertices (nodes) are the kinds of commodity $\mathbf{i}_\alpha$ appearing in the value-relation $\succ$.

- Join two nodes $\mathbf{i}_\alpha$ and $\mathbf{i}_\beta$ by an arrow (directed edge) $e_{\alpha\beta}$ from $\mathbf{i}_\alpha$ to $\mathbf{i}_\beta$ if and only if there exist $x_\alpha > 0$ and $x_\beta > 0$ such that $x_\alpha \mathbf{i}_\alpha \succ x_\beta \mathbf{i}_\beta$.

- Define the weight of edge $e_{\alpha\beta}$ to be $w_{\alpha\beta} = x_\alpha / x_\beta$.

2.5.2 Total and general value-forms

Economically unrealistic conditions satisfying our definition of a value-relation $\succ$ can easily be dreamed up. For instance, suppose we allow a *cycle* to appear in $\Delta(\succ)$:

$$
\Delta\left(\left\{ x\mathbf{a} \succ y\mathbf{b}, z\mathbf{b} \succ w\mathbf{c}, u\mathbf{c} \succ v\mathbf{a} \right\} \right)
$$
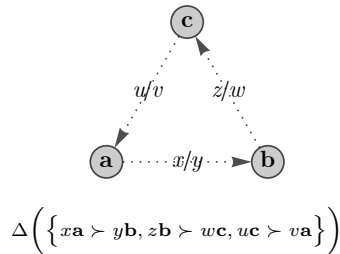
If we take $v = 2u$, $w = 2z$, and $y = 2a$, then $x\mathbf{a}$ can be traded for $8x\mathbf{a}$ through the series of exchanges

$$
x\mathbf{a} \longleftrightarrow 2x\mathbf{b}, \qquad 2x\mathbf{b} \longleftrightarrow 4x\mathbf{c}, \qquad 4x\mathbf{c} \longleftrightarrow 8x\mathbf{a}.
$$

In the absence of any generalized conditions governing the exchange ratios encoded in the relation $\succ$, such scenarios are possible formally, and may even be materially realizable, e.g. in trade between largely independent national

Figure 3: Visualization of a total value-form based on C1, 154-155.

RELATIVE	$\underset{\text{is worth}}{\succ}$	EQUIVALENT
$20\mathbf{l} = 20$ yd. linen		1 coat $= 1\mathbf{c}$
20 yd. linen		10 lb. tea $= 10\mathbf{t}$
20 yd. linen		40 lb. coffee $= 40\mathbf{j}$
20 yd. linen		1 qr. corn $= 1\mathbf{m}$
20 yd. linen		2 oz. gold $= 2\mathbf{g}$
20 yd. linen		$\frac{1}{2}$ ton iron $= \frac{1}{2}\mathbf{i}$

$$\mathfrak{M} = \{20\mathbf{l}, 1\mathbf{c}, 10\mathbf{t}, 40\mathbf{j}, 1\mathbf{m}, 2\mathbf{g}, \tfrac{1}{2}\mathbf{i}\}$$

$$\Delta\left(\left\{20\mathbf{l} \succ 1\mathbf{c}, 20\mathbf{l} \succ 10\mathbf{t}, 20\mathbf{l} \succ 40\mathbf{j}, 20\mathbf{l} \succ 1\mathbf{m}, 20\mathbf{l} \succ 2\mathbf{g}, 20\mathbf{l} \succ \tfrac{1}{2}\mathbf{i}\right\}\right)$$
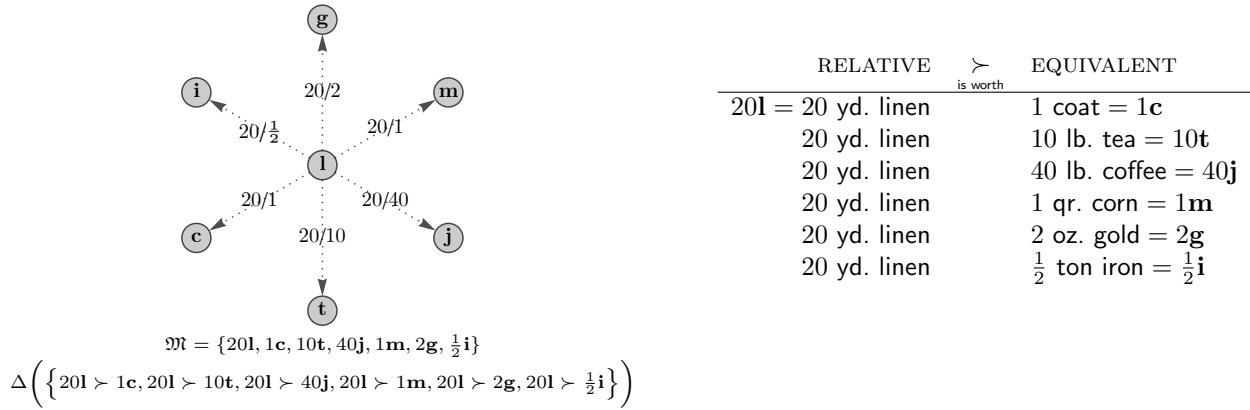
economies. In practice, however, we will reserve the term "value-form" for the two classes of value-relations which represent, so to speak, a *governing regime* of exchange—namely, total and general value-forms.

The simplest possible value-relation is defined on a universal set consisting of only two commodity-masses, $x\mathbf{i}$ and $y\mathbf{j}$:

$$\Delta\left(\{x\mathbf{i} \succ y\mathbf{j}\}\right)$$

Marx calls such a value-relation **accidental**.[48] It models isolated barter. It is ungovernable in the sense that society as a whole cannot have much to say about any actual exchanges made under such conditions, since the single given rate of exchange has nothing in common with the exchange of any other pair of commodities.

Definition 2.13. Given a market $\mathfrak{M}$, denote

$$\mathcal{K}(\mathfrak{M}) = \left\{\mathbf{i} \,\middle|\, x\mathbf{i} \in \mathfrak{M} \text{ for some } x > 0\right\}$$

and

$$E(\mathbf{i}) = \left\{\mathbf{j} \in \mathcal{K}(\mathfrak{M}) \,\middle|\, \mathbf{j} \neq \mathbf{i}\right\} \qquad \left(\mathbf{i} \in \mathcal{K}(\mathfrak{M})\right).$$

Definition 2.14. We define a **total value-form**[49] to be a value-relation $\succ$ defined on a market $\mathfrak{M}$ such that all elementary forms of value have some mass of a single kind of commodity as their relative form.

That is, a total value-form is a value-relation

$$\succ \; = \left\{x_{\mathbf{j}}\mathbf{i} \succ y_{\mathbf{j}}\mathbf{j} \,\middle|\, \mathbf{j} \in E(\mathbf{i})\right\}$$

for some fixed choice of $\mathbf{i} \in \mathcal{K}(\mathfrak{M})$. It is determined by a choice of $\mathbf{i}$ and masses $x_{\mathbf{j}}, y_{\mathbf{j}}$ for all $\mathbf{j} \in E(\mathbf{i})$.

---

[48] "The Simple, Isolated, or Accidental Form of Value [...] 20 yards of linen are worth 1 coat" (C1, 139)
[49] "The Total or Expanded Form of Value" (C1, 154)

Figure 4: Visualization of a general value-form based on C1, 157.

RELATIVE	$\succ$ is worth	EQUIVALENT
$1\mathbf{c} = 1$ coat		20 yd. linen $= 20\mathbf{l}$
$10\mathbf{t} = 10$ lb. tea		20 yd. linen
$40\mathbf{j} = 40$ lb. coffee		20 yd. linen
$1\mathbf{m} = 1$ qr. corn		20 yd. linen
$2\mathbf{g} = 2$ oz. gold		20 yd. linen
$\frac{1}{2}\mathbf{i} = \frac{1}{2}$ ton iron		20 yd. linen

$$\Delta\left(\left\{1\mathbf{c} \succ 20\mathbf{l}, 10\mathbf{t} \succ 20\mathbf{l}, 40\mathbf{j} \succ 20\mathbf{l}, 1\mathbf{m} \succ 20\mathbf{l}, 2\mathbf{g} \succ 20\mathbf{l}, \tfrac{1}{2}\mathbf{i} \succ 20\mathbf{l}\right\}\right)$$

Here we might imagine a society of smallhold farmers who are also part-time artisans, all of whom produce corn which they consume only in part, using the remainder as a means of exchange for use-values they do not themselves produce.[50] The corn that enters into exchange "is worth" whatever it's exchanged for. It *must* be exchanged in so far no artisan-farmer is able to produce all the various kinds of use-values they need for subsistence.

However socially realistic a total value-form may or may not be, such a protocol for exchange is in principle systemically governable. For example, climate conditions may decrease or increase the amount of labor-time required to produce corn, simultaneously altering all the exchange ratios between corn and other kinds of commodity.

Definition 2.15. We define a **general value-form**[51] to be a value-relation $\succ$ defined on a market $\mathfrak{M}$ such that all elementary forms of value have a given mass of a single kind of commodity, called the **universal equivalent**, as their equivalent form.
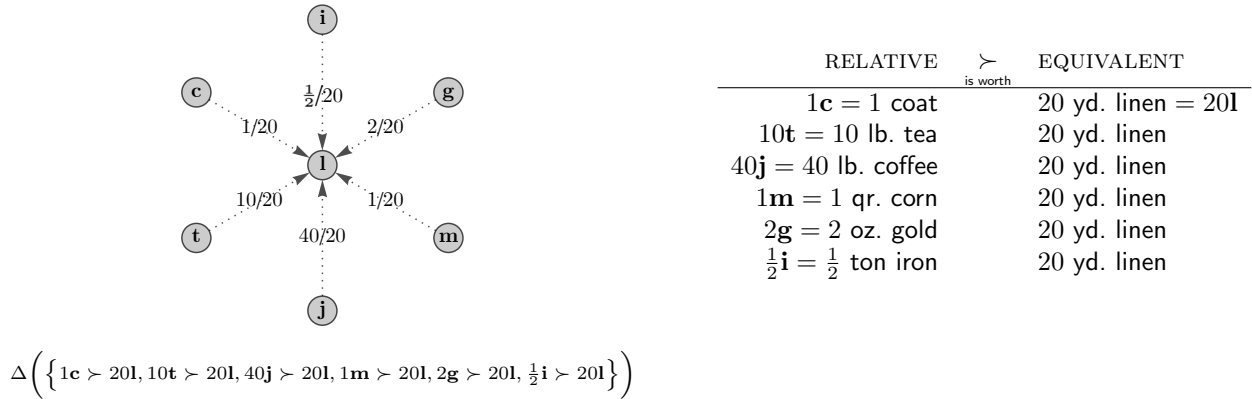
That is, a general value-form is a value-relation

$$\succ = \left\{x_{\mathbf{i}}\mathbf{i} \succ y_{\mathbf{i}}\mathbf{j} \,\middle|\, \mathbf{i} \in E(\mathbf{j})\right\}$$

for some fixed choice of (universal equivalent) $\mathbf{j} \in \mathcal{K}(\mathfrak{M})$. It is determined by a choice of $\mathbf{j}$ and masses $x_{\mathbf{i}}, y_{\mathbf{i}}$ for all $\mathbf{i} \in E(\mathbf{j})$.

Here every kind of commodity $\mathbf{a}$ must be exchanged for the universal equivalent $\mathbf{l}$ in order to acquire any third kind of commodity $\mathbf{b}$.

Definition 2.16. When the universal eqiuvalent is *currency*, a total value-form will be called a **money-form**.

---

[50] "The expanded form of value comes into actual existence for the first time when a particular product of labor, such as cattle, is no longer exceptionally, but habitually, exchanged for various other commodities." (C1, 158)

[51] "The General Form of Value" (C1, 157)

Figure 5: Marx's example of a money-form (C1, 162) as given (*left*) and with reduced ratios of exchange (*right*).
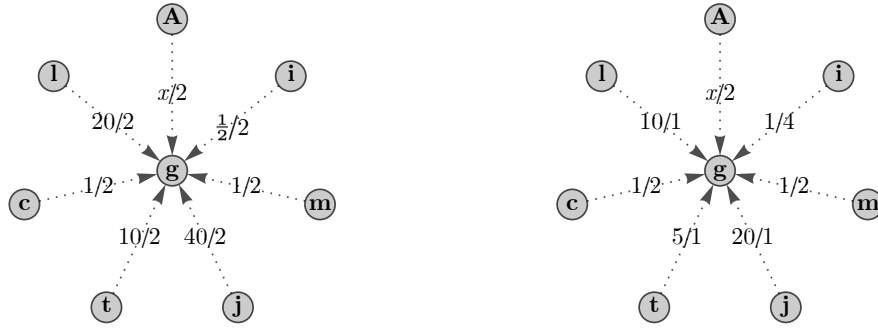
Example:

- Figure 5 gives the geometric realization of Marx's example of a money-form with unreduced and reduced ratios of exchange.
- The ratio of labor-time in the branches of production that produce coffee and tea is

$$\frac{L(x_\mathbf{t})}{L(x_\mathbf{j})} = \frac{x_\mathbf{j}}{x_\mathbf{t}} = \frac{40/2}{10/2} = \frac{20/1}{5/1} = 4;$$

that is, the workers who produce tea must work 4 times as long as the workers who produce coffee to produce a commodity-mass of equal value.

2.5.3 Relations of production encoded in the value-form

We summarize our formalization of the the value-form:

(a) The set $\mathcal{K}(\mathfrak{M})$ consists of the kinds of commodities $\mathbf{i}, \mathbf{j}, \ldots$ that can be exchanged at a given time and place.

(b) The relation $\succ$ specifies which kinds of commodity *must* be exchanged; these are the kinds of commodities $\mathbf{X} = x\mathbf{i}$ that appear in $\succ$ in the relative form.

(c) The amount x of $\mathbf{i}$ that must be exchanged, and the amount y of the commodity in the equivalent form $\mathbf{Y} = y\mathbf{j}$ it exchanges for, is specified by some elementary form of value $x\mathbf{i} \succ y\mathbf{j}$ in $\succ$.

(d) All such amounts (z for $z\mathbf{k} \in \mathfrak{M}$) are proportionate to the amount of socially necessary abstract labor embodied in—or equivalently, the expenditure of labor-power required to produce—the corresponding commodity-mass.

(e) Two commodity-masses are equal in value $\cong$ if and only if there exists some finite sequence of fair exchanges—a *path*

$$\mathbf{X}_1 \longleftrightarrow \mathbf{X}_2 \longleftrightarrow \cdots \longleftrightarrow \mathbf{X}_k$$

—such that either $\mathbf{X}_i \succ \mathbf{X}_{i+1}$ or $\mathbf{X}_{i+1} \succ \mathbf{X}_i$ for each exchange in the sequence.

Our definition of the value-form thus encodes

(a) the historically determined diversity of use-values, localized in time and place,

(b) the power relation between those who *must* exchange and those who do not need to, as expressed by the asymmetric value-relation $\succ$ between relative and equivalent forms,

(c) the division of labor as expressed by the proportions in which relative and equivalent forms exchange,

(d) the society's collective objectification of human labor through commodification, i.e. the transformation of use-value into *exchange-value* = value modulo use-value, and

(e) commodity fetishism in its apparent phenomenal form, by which we mean a relation $\cong$ that has the properties of numerical equality (reflexivity, symmetry, transitivity) on the basis of which value can be misconceived as a *numeric, one-dimensional* "exchange-value," e.g. price or marginal demand. To the extent that the analysis of value is carried out based on the properties of numerical equality (value of $x\mathbf{a}$) = (value of $y\mathbf{b}$), the *relational* properties of value—and in particular the asymmetry of the value-relation $\succ$ through which is enforced much of the coercion and exploitation inherent in the capitalist mode of production—are formally forgotten,[52] "modded out," along with (in the conceptual transformation of use-values into commodities) utility with regard to human wants.

On the level of individuals, the relative form *must* be exchanged; the equivalent form is what it must be exchanged for. If I must exchange a thing in order to realize its value, its worth must not consist in its ability to satisfy my human wants—I do not need it. Therefore, the relative form can only be occupied by a surplus, that is, a commodity-mass I possess in excess of the amount I need. Conversely, any surplus I have, I am able to exchange. Thus commodity-masses which appear in the relative form are exactly those which for me constitute a surplus of use-value. The economic reason I must exchange the relative value in order to realize its worth is that, for me, its use-value is nil, i.e., it is a surplus.

At the societal level, we claim that the relative form is characterized by the potential production or possession of a surplus, as opposed to the equivalent form, for which a surplus is *socially impossible*—e.g., you can never have too much money (the universal equivalent of the money-form).

To see this, first consider the total value-form, formally the opposite of the money-form, for which we had the example of a society of artisan-farmers who all produce some crop, say corn, and who all also produce various hand-manufactured useful items in part for personal use and in part for exchange.

Here the relative form is universal; corn must be exchanged for every other kind of commodity. It is possible to

---

[52] A mathematical **functor** transforms an object belonging to one abstract category into an object belonging to a second (possibly the same) category, translating the elements of the source object—and the relations between those elements—into elements of the target object and relations between the transformed elements. (We omit the details.) Given an object in some abstract category, let us call its elements, relations, and their respective properties its **structure**. A functor is **forgetful** if the structure is not entirely preserved in the transformation to the target object. The conceptual operation of "setting aside certain differences," which can be formally expressed in mathematics as *modding out*, is an example of a forgetful functor. The conceptual passage from the asymmetric value-relation $\succ$ to the symmetric value-relation $\cong$ can be visualized and formally expressed as a forgetful transformation of a directed graph (network diagram in which the segments joining two nodes are arrows) to an undirected graph (replace the directed arrows with undirected line segments).

have a surplus of corn—in fact, it's necessary, in practice, since without corn, one could not trade for necessities produced by other artisan-farmers.

The goods one does not produce, on the other hand, are scarce from the individual's perspective. In that sense one can never have too many sawblades or yards of fine fabric; but one *must* always have on hand an excess of corn to dispose of.

Now we reverse the arrows and obtain a general value-form with corn as the universal equivalent. The idea of "too much corn" is no longer conceivable, while every other kind of commodity can be produced to a surplus.

If we imagine that such a change has taken place in our artisan-farmer community, it must be that no surplus of self-produced corn is accessible to the community's members, however that circumstance came to pass. For if one can create a surplus of corn by one's own labor, then there is no need to exchange for it, contradicting that corn is the equivalent form; conversely, if one *does* need to trade for corn, then one must not be capable of independently producing an adequate amount of the stuff.

As for commodities other than corn, it is now possible to produce them in surplus amounts. In fact one is *compelled* to, exactly to the extent that one produces an amount of corn less than or equal to their personal need.

It's possible to "break even," in the sense that exactly enough corn is produced to sustain oneself. Even still, one must produce a surplus of sawblades, fine linen—anything other than corn—since an artisan-farmer cannot survive on corn alone.

Compare now the latter situation with life under capitalism, assuming a metal currency for simplicity. Gold is the universal equivalent. One cannot have too much of it. All kinds of commodities other than gold must be produced to excess, since one needs money to trade for commodities other than gold,[53] and since no one, owners of literal gold mines notwithstanding, produces all the commodities they need.

Now, the *workers*, in so far as they do not possess the means of production, cannot be said to produce *any* kind of commodity *except in cooperation with the owner classes*. It is possible to produce sawblades to excess, but of course not every member of the society can do it unaided. The possibility of producing a surplus of commodities exchangeable for money is a nonfiction only for those who hold the material means of producing it.

Nonetheless, class structure is not explicitly coded into the value-form. It is easy to imagine very different societies governed by essentially the same value-form, as our examples were intended to illustrate. When we say that a surplus of the equivalent form is socially impossible, we mean to indicate the circumscribed possibilities *for those with the greatest relative economic advantages in the society*, whether this refers to a tiny few, in the case of the autocracy of capital, or the majority, in the case of certain types of commune.

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[53] "Just like any other producer, [the capitalist] has first to transform the commodity into money by selling it, before he can manipulate it any further; he must convert it into the form of the universal equivalent." (C2, 463)

3 Production

3.1 Calculation of exchange-value of labor-power

We now compute the exchange-value of labor-power under a money-form, say with universal equivalent $\mathbf{g} = £1$ worth of gold.

Write $\mathbf{L} = 1$ hr labor-power. Assuming that labor-power can be exchanged, $x\mathbf{L} \succ y\mathbf{g}$ for some $x > 0$ and $y > 0$ by definition of a money-form.

How is y determined? We have $L_{\mathbf{L}}(x) = L_{\mathbf{g}}(y)$ by Axiom 3, so $L_{\mathbf{L}}(1)/L_{\mathbf{g}}(1) = y/x$ by Axiom 2, where

$$L_{\mathbf{g}}(1) = \text{amount of labor-power needed to produce } 1 \text{ unit of gold}$$

and

$$L_{\mathbf{L}}(1) = \text{amount of labor-power needed to produce } 1 \text{ hr of labor-power.}$$

The meaning of $L_{\mathbf{g}}(1)$ is clear. But what does it mean to "*produce*" an hour of labor-power?

Labor-power consists of "human brains, muscles, nerves, hands etc." (C1, 134), measured in units of time. To produce labor-power means to sustain human life in a state fit for work with the usual caveats—at an *average* rate of productivity, under *average* conditions, etc. To produce x hr of labor-power therefore means to provide for x hours of the worker's subsistence.[54] In essence, $L_{\mathbf{L}}(x) = x\,L_{\mathbf{L}}(1)$ is a *minimum* or *"living" wage* for x hours, since to under- or overprovide for the workers' average needs would result in extraordinary working conditions.

Clearly $L_{\mathbf{L}}(x)$ cannot exceed x as a matter of course. For if this were the case, then to provide for x hr of labor-power would require more time than the amount of productive available labor-time thus yielded. For example, if a society demands the expenditure of a full 24 hr of labor-power in order to meet the daily total social need, then by assumption $L_{\mathbf{L}}(24) > 24$, i.e. more hours of labor-power than there are in a day would first need to be expended in order to meet the daily need, which is not a recipe for a flourishing society.

Axiom 4. $L_{\mathbf{L}}(1) \leq 1$.

---

[54] "The value of labor-power is determined, as in the case of every other commodity, by the labor-time necessary for the production, and consequently also the reproduction, of this specific article. [...] Labor-power exists only as a capacity of the living individual. Its production consequently pre-supposes his existence. Given the existence of the individual, the production of labor-power consists in his reproduction of himself or his maintenance. For his maintenance he requires a given quantity of the means of subsistence. Therefore the labor-time requisite for the production of labor-power is the same as that necessary for the production of those means of subsistence; in other words, the value of labor-power is the value of the means of subsistence necessary for the maintenance of the laborer. However, labor-power becomes a reality only by its exercise; it is activated only by working. But in the course of this activity, i.e. labor, a definite quantity of human muscle, nerve, brain, etc. is expended, and these things have to be replaced. Since more is expended, more must be received. (*Marx's note*: In ancient Rome, therefore, the *villicus*, as the overseer of the agricultural slaves, received 'more meager fare than working slaves, because his work was lighter' [...]) If the owner of labor-power works today, tomorrow he must again be able to repeat the same process in the same conditions as regards health and strength. His means of subsistence must therefore be sufficient to maintain him in his normal state as a laboring individual. His natural needs, such as food, clothing, fuel, and housing, vary according to the climatic and other physical peculiarities of his country. On the other hand, the number and extent of his so-called necessary wants, as also the manner in which they are satisfyied, are themselves the product of historical development, and depend therefore to a great extent on the degree of civilization attained by a country; in particular they depend on the conditions under which, and consequently on the habits and expectations with which, the class of free laborers has been formed. In contrast, therefore, to the case of other commodities, there enters into the determination of the value of labor-power a historical and moral element. Nevertheless, in a given country, at a given period, the average quantity of the means of subsistence necessary for the laborer is a known *datum*." (C1, 274-275)

Suppose now that equal amounts of labor-power, say 24 hr, must be expended to produce $x = 48$ hr of labor-power and to produce the amount of gold represented by $y = 12$ shillings (s. $= \frac{1}{20}\pounds$), that is,

$$L_{\mathbf{L}}(48) = 24 = L_{\mathbf{g}}(\tfrac{12}{20}). \tag{$*$}$$

By Proposition 1,

$$t\mathbf{L} \cong \tfrac{1}{80}\, t\mathbf{g} = \tfrac{t}{4} \text{ s.}$$

for any $t > 0$. For example, the exchange-value of, say, $12\mathbf{L} = 12$ hours of labor-power is 3 shillings:

$$12\mathbf{L} \cong \tfrac{3}{20}\mathbf{g} = 3 \text{ s.}$$

Now let us consider the value *created* by t hr of labor-power.

Write $\mathbf{w} = 1$ hr of completed work. Clearly $L_{\mathbf{w}}(t) = t$ for any $t > 0$, since t hr of labor-power is expended in the production of t hr of completed work. To continue with our example, from ($*$) we have

$$L_{\mathbf{w}}(12) = 12 = L_{\mathbf{g}}(\tfrac{6}{20})$$

by linearity, yielding

$$12\mathbf{w} \cong \tfrac{6}{20}\mathbf{g} = 6 \text{ s.}$$

Here we see the formal origin of *surplus-value*: the value *created* by 12 hr of labor, $L_{\mathbf{w}}(12) = 6$ s., is greater than 3 s. $=$ what 12 hr of labor-power EXCHANGE FOR. The exact ratio of the value of the product to the value of the labor-power expended in production is given by

$$\frac{L_{\mathbf{L}}(t)}{L_{\mathbf{w}}(t)} = \frac{L_{\mathbf{L}}(t)}{t} = L_{\mathbf{L}}(1) = \left(\begin{array}{c} \text{amount of labor-power that must be} \\ \text{expended to produce 1 hr of labor-power} \end{array} \right).$$

Under a money-form, labor-power is in the relative form. This means first of all that labor-power *must* be exchanged in order to realize its worth. Secondly, it means that there must exist personal *surpluses* of labor-power, which is necessarily the case under the assumption that $L_{\mathbf{L}}(1) < 1$, i.e. the number of hours in a day exceed the number of hours we must labor in order to perpetuate ourselves and our society.[55] Such is indeed the condition of the proletarian classes, who are by definition deprived of the means to produce their own subsistence. The workers are *compelled* to exchange their labor-power on the assumption that a money-form governs exchange, and they are *able* to exchange their labor-power on the assumption that society is able to reproduce itself. The possibility of extracting surplus-value—described above as the excess of the value *created*, $L_{\mathbf{w}}(t)$, over and above

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[55] "In point of fact, capitalist production is commodity production as the general form of production, but it is only so, and becomes ever more so in its development, because labor itself here appears as a commodity, because the worker sells labor, i.e. the function of his labor-power, and moreover, as we have assumed, at a value determined by the costs of its reproduction. [...] In the relation between capitalist and wage-laborer, the money relation, the relation of buyer and seller, becomes a relation inherent in production itself. But this relation rests fundamentally on the social character of production, not on the mode of commerce; the latter rather derives from the former. It is typical of the bourgeois horizon, moreover, where business fills the whole of people's minds, to see the foundation of the mode of production in the mode of commerce corresponding to it, rather than the other way round." (C2, 196)

the value *of the labor-power expended*, $L_{\mathbf{L}}(t)$—cannot be eliminated under these assumptions, which imply that $L_{\mathbf{L}}(t) < t = L_{\mathbf{w}}(t)$ $(t > 0)$.

The worker sells their labor-power, *not their completed labor.* This fact assumes great importance in calculations, and also explains why "labor," *expended labor-power,* does not appear in the value-form. The function we have called $L_{\mathbf{w}}$, which gives the number of hours needed to produce x hours of completed work, is simply the identity map $t \mapsto t$. It shows up in calculations only where the value of a *finished product* is concerned. It does not in itself imply any exchange. It therefore does not need to appear in the value-form. The function $L_{\mathbf{L}}$, on the other hand, is what determines the exchange-value of labor-power, conceptually transforming masses of labor-power into exchangeable commodities. Under a money-form, the function $L_{\mathbf{L}}$ makes it possible to sell labor-power by measuring its value as a function of mass so that a given mass of labor-power can be exchanged with other commodity-masses to which it is equal in value ($=$ hours expended in production).

3.2   Marx's example in C1, Ch. 7: Spinning cotton into yarn

In this subsection we work through the example given by Marx in C1, Ch. 7.

Exchange-value of raw materials and gold

Suppose 10 lb. cotton has been bought for 10 shillings[56] (s.) and one whole spindle for 8 s.[57]

Further suppose that 24 hr labor (two 12 hr working-days) is required to produce the amount of gold represented by 12 s.[58]

We summarize what we've been given so far:

COMMODITY:	12 s. (gold)	10 lb. cotton	1 spindle
LABOR ABSORBED BY PRODUCTION:	24 hr		
IS WORTH ($\succ$):		10 s.	8 s.

Assume $\frac{1}{4}$ of a spindle is lost to wear and tear in the process of spinning 10 lb. cotton into 10 lb. yarn.[59] We see that the raw materials for spinning 10 lb. yarn can be obtained for 12 s.:

$$\begin{array}{ccccc}
\text{raw materials for 10 lb. yarn} & = & \text{10 lb. cotton} & + & \frac{1}{4}\text{ spindle} \\
\shortparallel & & \shortparallel & & \shortparallel \\
\text{12 s.} & = & \text{10 s.} & + & \text{2 s.}
\end{array}$$

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[56] "our capitalist has, we will assume, bought [the cotton] at its full value, say 10 shillings." (293)

[57] "We will further assume that the wear and tear of the spindle, which for our present purpose may represent all other instruments of labour employed, amounts to the value of 2 shillings." (293-4)

[58] "If, then, twenty-four hours of labour, or two working days, are required to produce the quantity of gold represented by 12 shillings, [...]" (294, quote continues)

[59] "For spinning the yarn, raw material is required; suppose in this case 10 lb. of cotton." (293)

Since

$$(\text{raw materials for 10 lb. yarn}) \cong 12 \text{ s.},$$

the amount of labor-power expended to produce raw materials for 10 lb. yarn is equal to

$$L_{\mathbf{g}}(12 \text{ s.}) = L_{\mathbf{g}}(\tfrac{12}{20}) = 24 \text{ hr,} \,^{[60]}$$

and similarly for any multiple thereof:

	raw materials for 10 lb. yarn	raw materials for 20 lb. yarn
LABOR ABSORBED BY PRODUCTION:	24 hr	48 hr
IS WORTH ($\succ$):	12 s.	24 s.

Absorption of labor by production

Assume that $\frac{5}{3}$ lb. cotton can be spun into $\frac{5}{3}$ lb. yarn in 1 hr. Then 10 lb. cotton can be spun into 10 lb. yarn in 6 hr.[61]

	$\frac{5}{3}$ lb. yarn	10 lb. yarn
LABOR ABSORBED BY PRODUCTION:	1 hr	6 hr

Value created by labor

Since

$$6\mathbf{w} \cong \tfrac{3}{20}\mathbf{g} = 3 \text{ s.},$$

a value of 3 s. is added to the cotton by labor in the production of 10 lb. yarn.[62]

	12 s.	10 lb. yarn
LABOR ABSORBED BY PRODUCTION:	24 hr	6 hr
VALUE CREATED BY PRODUCTION:		3 s.

It follows that "an adequate price" (297) for 10 lb. yarn is 15 s.:

	raw materials for 10 lb. yarn	10 lb. yarn
LABOR ABSORBED BY PRODUCTION:	24 hr	6 hr
VALUE CREATED BY PRODUCTION:		3 s.
IS WORTH ($\succ$):	12 s.	15 s.

---

[60] "[...] it follows first of all that two days of labour are objectified in the yarn." (294, quote continued)

[61] "If in one hour $1\frac{2}{3}$ lb. of cotton can be spun into $1\frac{2}{3}$ lb. of yarn, then 10 lb. of yarn indicate the absorption of 6 hr of labour." (297)

[62] "in 6 hours he will convert 10 lb. of cotton into 10 lb. of yarn. Hence, during the spinning process, the cotton absorbs 6 hours of labour. The same quantity is also embodied in a piece of gold of the value of 3 s. A value of 3 s., therefore, is added to the cotton by the labour of the spinning." (297)

"Our capitalist stares in astonishment. The value of the product is exactly equal to the value of the capital advanced." The serious student stares, too—and wonders, "where are the profits?" An academic discussion ensues, and perhaps grows heated. But meanwhile, Marx continues,

> after a hearty laugh, [the capitalist] re-assumes his usual mien. Though he chanted to us the whole creed of the economists, in reality, he says, he would not give a brass farthing for it. He leaves this and all such like subterfuges and juggling tricks to the professors of Political Economy, who are paid for it. He himself is a practical man; and though he does not always consider what he says outside his business, yet in his business he knows what he is about.

Extraction of surplus-value from labor

We find that the capitalist's revenue exceeds his costs. To see this, compare the value of labor-power plus the value of the raw materials, a total of 27 s.—

	raw materials for 20 lb. yarn	12 hr labor-power (1 working-day)	costs of production for 20 lb. yarn
LABOR ABSORBED BY PRODUCTION:	48 hr	6 hr	60 hr
VALUE CREATED BY PRODUCTION:			6 s.
IS WORTH ($\succ$):	24 s.	3 s.	27 s.

—with the the value of the finished product:

	raw materials for 20 lb. yarn	20 lb. yarn
LABOR ABSORBED BY PRODUCTION:	48 hr	12 hr
VALUE CREATED BY PRODUCTION:		6 s.
IS WORTH ($\succ$):	24 s.	30 s.

The following definitions include notation consistent with §.

Definition 3.1.

- C' = **exchange-value of product** = exchange-value of raw materials + exchange-value added by labor
- C = **exchange-value of production costs** = exchange-value of raw materials + exchange-value of labor-power consumed

3.3   Linear model for the production of value as a function of labor-time

Let us now state an elementary linear model for the production of value as a function of labor-time.

In it will be represented (1) the "inputs" to the production process, including labor-power, (2) labor-time, and (3) the "outputs," that is, the quantity of commodity and the surplus-value produced.

Let $\mathbf{x}$ be a vector representing the production process, called the **production** (or **process**) **vector**, defined as follows:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \text{raw material} \\ \text{wear and tear} \\ \text{1 hr labor-power} \\ \text{commodity produced} \\ \text{surplus-value} \end{bmatrix} \begin{array}{l} \leftarrow \text{units per labor-hour consumed} \\ \leftarrow \text{units per labor-hour of implement consumed} \\ = \text{units per labor-hour of labor-power expended} \\ \leftarrow \text{units per labor-hour of finished product produced} \\ \leftarrow \text{value per labor-hour extracted} \end{array}$$

Let R be the diagonal matrix, called the **exchange** (or **exchange-rate**) **matrix**, with diagonal entries r_k ($1 \leq k \leq 5$), where

$$r_1 = \frac{y_1 \text{ shillings}}{x_1 \text{ units raw material}}, \qquad r_2 = \frac{y_2 \text{ shillings}}{x_2 \text{ instruments}}, \qquad r_3 = \frac{y_3 \text{ shillings}}{x_3 \text{ hr labor-power expended}},$$

$$r_4 = \frac{y_4 \text{ shillings}}{x_4 \text{ units produced}}, \qquad r_5 = 1,$$

and the denominator of each fraction $r_i = \mathbf{V}_i / \mathbf{U}_i$ satisfies $\mathbf{U}_i \succ \mathbf{V}_i$ ($1 \leq i \leq 4$).

By definition, surplus-value x_5 is equal to the value of the finished product, minus the value of all inanimate materials consumed (the "means," or the "objects" and "instruments," of production), minus the value of the human labor-power expended:

$$
\underbrace{\overbrace{x_5 = \underbrace{r_4 x_4}_{\substack{\text{value of} \\ \text{finished} \\ \text{product}}} - \underbrace{\overbrace{\underbrace{r_1 x_1}_{\substack{\text{value of} \\ \text{objects} \\ \text{of labor}}} - \underbrace{r_2 x_2}_{\substack{\text{value of} \\ \text{instru-} \\ \text{ments of} \\ \text{labor}}}}^{\text{means of production}}}}^{\text{forces of production}} - \underbrace{r_3 x_3}_{\substack{\text{value of} \\ \text{labor-} \\ \text{power} \\ \text{ex-} \\ \text{pended}}}}_{\text{capitalist mode of production}}
$$

For simplicity, we take all quantities on the right-hand side of the previous equation to be constants, e.g. we assume no change in productivity of labor, supply and demand, etc.

☞ In the foregoing example, we would have (omitting units on the right-hand sides of the equations expressing

6**x**, 12**x**, and 24**x** for readability's sake)

$$\mathbf{x} = \begin{bmatrix} \frac{5}{3} \text{ lb. cotton} \\ \frac{1}{24} \text{ spindle} \\ 1 \text{ hr labor-power} \\ \frac{5}{3} \text{ lb. yarn} \\ \frac{1}{4} \text{ shillings (s.)} \end{bmatrix}, \qquad 6\mathbf{x} = \begin{bmatrix} 10 \\ \frac{1}{4} \\ 6 \\ 10 \\ \frac{3}{2} \end{bmatrix}, \qquad 12\mathbf{x} = \begin{bmatrix} 20 \\ \frac{1}{2} \\ 12 \\ 20 \\ 3 \end{bmatrix}, \qquad 24\mathbf{x} = \begin{bmatrix} 40 \\ 1 \\ 24 \\ 40 \\ 6 \end{bmatrix},$$

$$R = \begin{bmatrix} \dfrac{20 \text{ s.}}{20 \text{ yd. cotton}} & 0 & 0 & 0 & 0 \\ 0 & \dfrac{8 \text{ s.}}{1 \text{ spindle}} & 0 & 0 & 0 \\ 0 & 0 & \dfrac{3 \text{ s.}}{12 \text{ hr labor-power}} & 0 & 0 \\ 0 & 0 & 0 & \dfrac{30 \text{ s.}}{20 \text{ yd. yarn}} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

For any number of working hours t in a calendar day $(0 < t \leq 24)$, the components y_k $(1 \leq k \leq 5)$ of the vector $R\mathbf{x} \cdot t$,

$$y_1(t) = r_1 x_1 t, \qquad y_2(t) = r_2 x_2 t, \qquad y_3(t) = r_3 x_3 t, \qquad y_4(t) = r_4 x_4 t, \qquad y_5(t) = x_5 t$$

give the value $y_1 + y_2$ of the objects and instruments of labor consumed, the value y_3 of the labor-power expended, the value y_4 of the commodities produced, and the surplus-value y_5 extracted from t hours of production in one calendar day, assuming supply, demand, productivity of labor, etc., fixed.

☞   E.g., writing $\mathbf{y}(t) = \big(y_1(t), \ldots, y_5(t)\big)$, $y_1 = y_1(t)$, $y_2 = y_2(t)$, etc.,

$$y_1 + y_2 + y_3 + y_5 - y_4 = 0; \qquad \mathbf{y}(12) = R\mathbf{x}(12) = \begin{bmatrix} 20 \text{ s.} \\ 4 \text{ s.} \\ 3 \text{ s.} \\ 30 \text{ s.} \\ 3 \text{ s.} \end{bmatrix}, \qquad \mathbf{y}(24) = R\mathbf{x}(24) = \begin{bmatrix} 40 \text{ s.} \\ 8 \text{ s.} \\ 6 \text{ s.} \\ 60 \text{ s.} \\ 6 \text{ s.} \end{bmatrix}.$$

3.4   Surplus-seeking behavior

A mass of capital in commodity-form is said to undergo **valorization** (*Kapitalverwertung*),[63] or is said to **valorize itself**, when its value increases due to the application of labor.

- Since human labor can act only on commodities, and not on capital in the monetary form, a mass of capital that undergoes valorization (*verwertet sich*) must necessarily enter into production in commodity form.

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[63] In plain German, *Verwertung* means the productive use of a resource so that it yields earnings.

- The mass of capital which is to valorize itself enters into production as a collection of inanimate inputs—the raw materials and instruments of labor (tools, infrastructure, etc.). Let us denote the total value of the inanimate inputs to production by m_p (for *means of production*). In Ch. 7's example, where so much cotton and so much of a spindle is consumed in the spinning of so much yarn, m_p is equal to the sum of the values of the yarn and the portion of the spindle consumed in production.

- Let C' denote the industrial output of production. That is, C' is the commodity-mass produced by the application of human labor, which we denote symbolically by $\rightsquigarrow$. The value of C' consists of m_p, plus the amount of value $L_{\mathbf{w}}(t) = t$ created by human labor during the production of C', where x is the number of hours of labor-power expended in the production of C'.

- Suppose $x\mathbf{L} \succ y\mathbf{g}$, that is, x hr labor-power IS WORTH y units of money (gold). Then the value of the production input, which we will denote by C, is equal to $m_p + L_{\mathbf{g}}(\frac{y}{x}t)$ by Proposition 1.

Using the notation indicated, we may symbolize the process of valorization by means of industrial production as follows:

$$C \;=\; m_p + L_{\mathbf{g}}(\tfrac{y}{x}t) \qquad \rightsquigarrow \qquad m_p + t \;=\; C'$$

Production under capitalism is **surplus-seeking behavior**;[64] by this we mean economic activity the purpose of which is *to maximize privately owned surplus*, and not necessarily[65] to satisfy human wants.[66] We cannot delve into the theoretical explanation and all the practical ramifications of this observation at this time. But let us take a moment to identify the imprint of surplus-seeking behavior in the structure of the value-form.

In §2.5.2 we gave the interpretation that the value-form encodes a *social* tendency to produce a surplus of the relative form for the purpose of being exchanged, and a tendency on the part of *individuals* to hoard the equivalent form. Let us apply this interpretation to wage labor in a society governed by a money-form.

Labor-power is in the relative form. The equivalent form is cash. Indeed, the industrial and post-industrial worker always has a surplus of labor-power on hand; this just means that they are not forced by an inhospitable environment or by a lack of adequate technology to spend their every waking moment working to secure their survival.

On a larger scale, the aggregate of personal surpluses of labor-power forms an enormous mass. But no activity originating on the side of this enormous mass can be described as surplus-seeking behavior. It is an economic

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[64] "valorization is the purpose of the [production] process" (C2, 172) ◇◇◇ "The immediate production process of capital is its process of labor and valorization, the result of this process being the commodity product, and its determining motive the production of surplus-value." (C2, 427)

[65] "Simple reproduction is oriented by nature to consumption as its aim. Even though the squeezing out of surplus-value appears as the driving motive of the individual capitalist, this surplus-value—no matter what its proportionate size—can be used here, in the last analysis, only for his individual consumption. ¶ In so far as simple reproduction is also part of any annual reproduction on an expanded scale, and the major part at that, this motive remains alongside the motive of enrichment as such and in opposition to it. In the real world the matter appears more intricate, since the partners who share the loot—the surplus-value of the capitalist—figure independently of him as consumers." (C2, 487)

[66] "For capitalism is already essentially abolished once we assume that it is enjoyment that is the driving motive and not enrichment itself." (C2, 199) Moore and Aveling have: "capitalism is abolished root and branch by the bare assumption that it is personal consumption and not enrichment that works as the compelling motive."

disadvantage to have more of the relative form than one needs, *except to the extent one is able to exchange it for an equivalent.*

The terms "supply" and "demand" typically describe God's-eye phenomena, predicated of an entire market economy en bloc. But if we limit ourselves to the perspective of a single class, very different phenomena—and very different motivations—emerge. The "*workers' demand*" consists of their human wants: food, shelter, clothing, tobacco, etc. The "*workers' supply*" consists of the portion of their living labor-power available for sale. It is clearly disadvantageous for the workers if there is an oversupply of labor-power, e.g. if the work-week is lengthened. Neither is it particularly advantageous for the workers to accumulate a surplus on the side of their "demand," since this would mean having more of this or that commodity than one can use *without* the ability to make of the surplus a steady source of income, since for that, ownership of the means of production would be required.

Now consider the *capitalist's demand* and the *capitalist's supply*.[67] They cannot be understood in terms of satisfying human wants; the point is to amass a maximal surplus.[68] Marx identifies the output of production—that is, the mass of commodities produced by the application of labor—as the **capitalist's supply**; we denoted its value above by C'. The raw materials and labor-power purchased by the capitalist, he calls the **capitalist's demand**;[69] we called its value C. Since the capitalist's goal is to widen as much as possible the margin $s = C' - C$ between their supply and their demand,[70] it makes sense to call their behavior—valorization by means of industrial production— surplus-seeking: from their perspective, a glut of (the capitalist's) surplus is, strictly speaking, impossible.

4 Reproduction

4.1 Four dimensions of Marxian economic analysis

Let us now formally introduce four basic dimensions of Marxian economic analysis: *constant capital*, *variable capital*, *surplus-value*, and time. We will begin with constant capital (c) and time. (Here we are broadly following the schematic of C2, Ch. 20.)

When production begins, it does not begin from scratch. Factories built in previous years are still standing. Seeds harvested from previous crops are still lying in storage. Let us call the existing means of production (raw materials, tools, infrastructure, semi-finished goods, etc.) **constant capital**. No new labor needs to be expended to obtain

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[67] "The capitalist casts less value into circulation in the form of money than he draws out of it, because he casts in more value in the form of commodities than he has extracted in the form of commodities. In so far as he functions merely as the personification of capital, his supply of commodity-value is always greater than his demand for it. If his supply and demand matched one another [...] this would be equivalent to the non-valorization of his capital; it would // not have functioned as productive capital; productive capital would have been transformed into commodity capital that had not been impregnated with surplus-value; it would not have extracted from labor-power through the production process any surplus-value in the commodity form, and thus not functioned as capital at all." (C2, 196-197)

[68] "[The capitalist's] goal is not simply to cover his demand with his supply, but to have the greatest possible excess of supply over demand. ¶ What is true for the individual capitalist, is also true for the capitalist class." (C2, 197)

[69] Our notation here is not consistent with the convention in later sections that $C = c + v$. Marx explicitly says that the capitalist's demand is bounded above by $c + v$ with equality only if the workers do not hoard (C2, 197).

[70] "The greater the difference between the capitalist's supply and his demand, i.e. the greater the additional commodity value that he supplies over the commodity value that he demands, the greater the rate at which he valorizes his capital." (C2, 197)

it; it is "dead" objectified labor which exists in usable form before the current period of production begins.

Let's assume that at the beginning of each period, we have a mass of capital, entirely in commodity form; let's say each period lasts a year; and let's say that the period ends when the mass of commodities has been replenished through production, the previous year's batch of commodities having entirely been sold in order to finance this year's production process.

In the sense that unfinished lumber no longer exists once it has been transformed into finished furniture, the material form of some portion of the constant capital is functionally destroyed by the process of production. It is consumed by the operation of labor. Labor transfers the value of this consumed portion of the constant capital to the commodities being produced. The value of the product thus decomposes into two components, the value c of the constant capital (dead labor accumulated in previous years) which is consumed in production this year, and the value $v + s$ of the labor-power expended in production.

The constant capital can be decomposed into **fixed** and **circulating** constant capital (although the classification of certain articles as fixed or circulating depends on context and is in some cases arbitrary). This distinction is somewhat technical. For example, an accounting must be made of the *depreciation value* transferred to the product over the life of, say, a machine in a factory.[71] We will simplify matters by decomposing the constant capital into *the portion used up during the current period of production*, on the one hand, and *the portion that appears in the following period as constant capital*, as Marx does when discussing the formation of the general rate of profit.[72]

The value of the component $v + s$ is proportional to the amount of labor expended during the current period of production. It further decomposes into the **variable capital** v and the **surplus-value** s.

The variable capital consists of the value of the means of subsistence needed to keep the workers alive and capable of doing work. In order to sustain systemic capitalism, the entire sum of the variable capital collectively possessed by the capitalists must be replenished, since it must be laid out over the course of the year in monetary form as **wages** to the workers, who use it to purchase means of subsistence. The replenishment of wages laid out by the collective capitalist is accomplished by the sale of the total product that the workers produce.

Now, the sale of the total product *must* result in a surplus above and beyond the minimum required for the subsistence of the workers. If the process of production did *not* produce such a surplus, the capitalist would have long ago abandoned the enterprise. Given a sufficiently developed level of technology and adequate natural

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[71] C2, §20.11
[72] C3, Ch. 9, 255-256

resources, the workers would only need to work a few hours a week—were they permitted to do so—in order to meet their material needs. But in that case, the capitalist would gain nothing. The capitalist therefore coerces the workers into working additional hours ("surplus labor"). The value s ("surplus-value") of the corresponding portion of the product ("surplus product") comprises the mass of the capitalist's profit in commodity form. Upon the sale of the total product, the monetary equivalent of v ends up (temporarily) in the hands of the workers, and the monetary equivalent s ends up (indefinitely) in the hands of the capitalist.

Let us assume for simplicity that:

- all of c is used up (hence must be replenished) in each period

- all of the surplus product from the previous period is withdrawn from circulation

Then our general schematic for annual reproduction, beginning and ending with a mass of commodities, is:
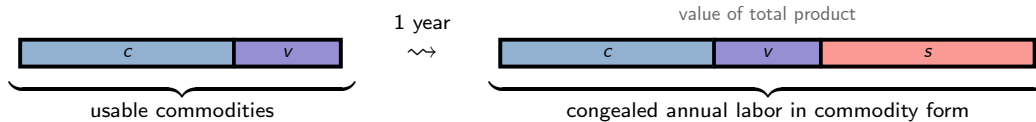

This periodic **circuit of commodity capital**[73] can be broken down into three successive stages:

1. (*Realization*, $C' - M'$:) A given mass C' of commodities no greater in value than $c + v$ is sold,[74] where $c =$ value of means of production consumed in each period of production, and $v =$ value of the necessary product. For simplicity, we assume C' is equal in value to $c + v$, and we disregard any surplus product produced in the previous period.
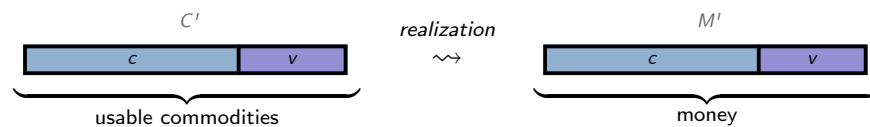
As a practical matter, in the absence of credit, a liquid reserve fund for repairs and modernization must be accumulated, so some portion of the surplus product from the previous period (which we are entirely ignoring here) must be sold.[75]

2. (*Advance*, $M' - \begin{bmatrix} L \\ mp \end{bmatrix}$:) Means of production and labor-power equal in value to c and to v, respectively, are purchased.
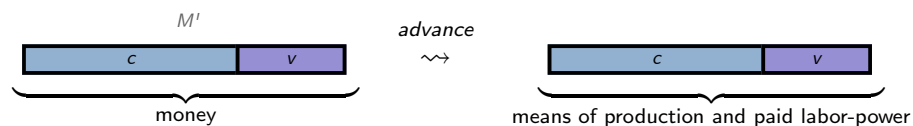
[73] C2, Ch. 3
[74] C2, 197
[75] C2, 199

3. (*Production*, $\left[\begin{smallmatrix} L \\ mp \end{smallmatrix}\right] \cdots P \cdots C''$:) The workers produce a mass of commodities equal in value to $c + v + s$, the sum of the constant capital c consumed and the labor-power $v + s$ expended in production. The portion of the mass of commodities corresponding to v is the necessary product (the result of paid labor) and the portion of the mass of commodities corresponding to s is the surplus product (the result of unpaid labor).
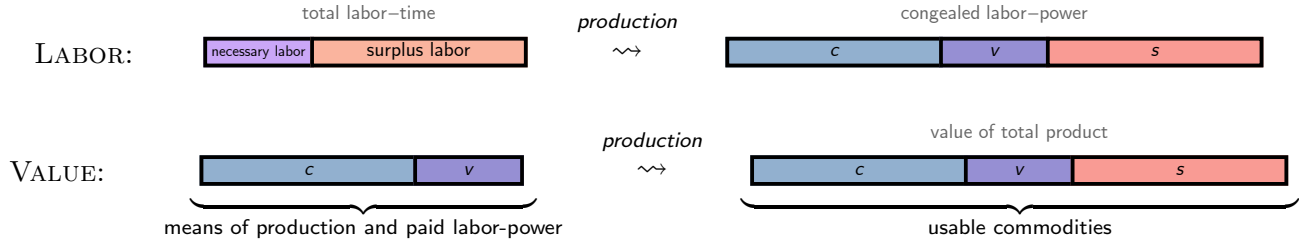
Marx describes the resulting mass of commodities (C'', equal in value to $c + v + s$) as **the capitalist's supply**,[76] commenting that "[w]hat is true for the individual capitalist, is also true for the capitalist class":[77] "[h]e sells dearer, not because he sells above the value of his commodities, but because he sells commodities of a value greater than the sum of the ingredients required to produce them."[78]

4.2 Capitalists' supply

5 Market prices and the general rate of profit

5.1 Functional definition of capital

Let us justify the pat (but not inapt) phrase, *"Capital is money that begets money."*

First we recall that, over the course of a chain of transactions and transformations, a single mass of capital undergoes changes in form. We will continue to follow the circuit of productive capital as it transforms from (1) commodity form to (2) its monetary expression, then back to (3) commodity form through the process of production ($C' - M' - \left[\begin{smallmatrix} L \\ mp \end{smallmatrix}\right] \cdots P \cdots C''$ being Marx's elaboration of the cycle $C' - \cdots - C''$ identified by Quesnay[79]).

To distinguish useful items (use-values) that function as **capital** from those that do not, the following two properties suffice:

 (i) capital is not consumed, and

(ii) an equal profit is expected upon all parts of the capital advanced.

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[76] C2, 198
[77] C2, 197
[78] C2, 197
[79] C2, 179

Based on this definition, we can clearly identify the economic role played by workers' wages. It is revenue equal in amount to the worker's consumption except to the extent that the worker is able to invest, i.e. to employ their revenue as capital.

In so far as the workers' wages are equal to the value of their means of subsistence, wages are purely consumed and thus by definition cannot function as capital; to the extent that the workers' wages are consumed and not invested, they do not constitute capital; to the dollar amount of surplus-value redistributed to the workers in excess of their means of subsistence, the workers collectively become capable of investing—at least in theory.

In practice, conditions of austerity and precarity compel workers to hoard, to accumulate, but this accumulated revenue consitutes only a buffer, a cache, a reserve fund *destined for consumption*. Just as the industrial capitalist hoards (in the absence of credit) in order to survive as a capitalist—that is, in order to maintain the ability to advance capital—so too must the workers (likewise assuming the absence of credit) hoard in order to reproduce their ability to fulfill their economic role—that is, in order to survive as workers, a predominantly non-investing, non-capitalist class. Only credit enables the individual to dispense with the precaution of accumulating a hoard. To the extent they are able, most often through loans, workers can and do invest, e.g., in real estate, or in their own education. But those investments cannot be called capital to the extent that their value is ultimately consumed.

In general, Property *(i)* ensures not just for the workers but also for the capitalists that capital must *constantly* reproduce itself, at the bare minimum; otherwise it becomes, in part or in its entirety, a fund for consumption, and therefore not capital.

We now show that Property *(ii)* is violated by the assumption that two masses of capital breed unequal amounts of surplus. Suppose that the total social capital A is divided into capitals I and II with respective rates α and β of profit from sales. Assume $\alpha < \beta$—say, $\alpha = 5\%$ and $\beta = 15\%$. Then an advance of \$1000 results in \$100 less profit for I than for II. This situation is indistinguishable from a situation in which an equal rate of profit $\beta = 15\%$ is applied to all of the capital II and exactly one-third of the capital I, since $15\% \cdot \frac{1}{3}\text{I} = \beta \cdot \frac{\alpha}{\beta}\text{I} = \alpha\text{I}$. Thus the assumption that $\alpha < \beta$ is equivalent to the assumption that $1 - \frac{\alpha}{\beta}$ of the capital I is unproductive. But this contradicts the definition of capital as *value upon the whole of which a return is expected* (*ii* above).

Property *(i)* requires only that no part of the capital be destroyed. It does not imply that the capital's value must increase, or even that it remains constant. It is obviously possible for the worth of an item to decrease for reasons other than it being consumed.

Property *(ii)*, on the other hand, characterizes capital as *infinitely expansible*, in principle.

5.1.1   Marx's sources for the definition of capital

Before resuming the larger argument, we note the citations Marx gives for the definition of capital.

According to Adam Smith,

> "[A man's] whole stock [...] is distinguished into two parts. That part which, he expects, is to afford him [...] revenue, is called his capital. The other is that which supplies his immediate consumption [...]" (Smith, *Wealth of Nations*, 1776).

Malthus puts the matter more starkly:

> "The capitalist . . . *expects* an equal profit upon all parts of the capital which he advances." (Malthus, *Principles of Political Economy*, 1836, as quoted in Marx, *Capital*, Vol. 3, Ch. 1, Marx's italics)

These definitions agree qualitatively with those found in widely available "orthodox" capitalist economic primers, such as the following excerpt from an explainer page on Investopedia.com:

> "The capital of a business is the money it has available to pay for its day-to-day operations and to fund its future growth. [...] In a broader sense, the term may be expanded to include all of a company's assets that have monetary value, such as its equipment, real estate, and inventory. But when it comes to budgeting, capital is cash flow. [...] *Capital is used by companies to pay for the ongoing production of goods and services in order to create profit.* [...] Labor and building expansions are two common areas of capital allocation. *By investing capital, a business or individual seeks to earn a higher return than the capital's costs.*" (M. Hargrave, *Investopedia.com*, "Capital," retrieved Mar. 10, 2021; italics added)

Property *(ii)* exactly expresses the characteristic quantitative relation between capital and the profit it breeds described above by Malthus and Investopedia. In terms that will be familiar to high-school and early-college math students, a mass of capital begins its growth cycle in the form of a **principal** P—or, to use a term that is more expressive of its function, an **initial advance**—which is invested or loaned for some period of time, at the end of which the value of the mass is called its **future value** A. Writing $s = I$ for interest in the usual formula for the future value of a loan or investment yields:

$$A = P + s.$$

The term s is the **amount of profit** returned to the owner of the initial advance. Property *(ii)* can now be formally stated by saying that, for any portion a of the initial advance A, the return σ on a is proportional to the return s on the entire amount A in the ratio a/A:

$$\frac{\sigma}{s} = \frac{a}{A} = \text{const.}$$

The term P aggregates—in Marx's terms—the constant capital and the circulating capital. In Quesnay's terms, P aggregates the *avancées primitives* and *avancées annuelles*. (Marx comments that Adam Smith improves upon

the Physiocrats when he translates *avancées* as *capital*.[80])

5.2   Capitalists' demand

5.3   Competition establishes a general rate of profit

While critiquing Ricardo's approach to the problem of reconciling value and the general rate of profit, Marx makes the following points:[81]

- the general rate of profit should not be naively assumed or asserted as part of the "law of value"

- instead, we should ask to what degree the hypothesis of a general rate of profit is consistent with the law that value is (exclusively) determined by labor-time

- on their face, these two propositions appear to contradict one another

- therefore, the existence of a general rate of profit can only be "explained through a number of intermediary stages"

6   Deviation of production price from value

6.1   Production supply and production demand

6.2   Relation of production supply to social labor-time

(C3, 288 ff.)

6.3   Production price almost always differs from value

(MECW 31, around 408)

6.4   Empirical correlation cases I–III

6.5   Production demand is error

for one thing, the discovery of new use-values 'is the work of history' (C1, Ch. 1)

when demand = supply, market price = market value (C3, 290 ff.)

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[80] C2, 438. See also C2, 435, 436, 444, 466.

[81] 'Instead of *postulating* this *general rate of profit*, Ricardo should rather have examined in how far its existence is in fact consistent with the determination of value by labor time, and he would have found that instead of being consistent with it, *prima facie,* it *contradicts* it, and that its existence would therefore have to be explained through a number of intermediary stages, a procedure which is very different from merely including it under the law of value.' (MECW 31, 401)

demand, supply, market price, market value, and production (C3, 292)

6.6 Structural determination of supply and demand

(C3, 294 ff.)

7 Capital conscious of itself as social power

8 Economic (synthetic) propositions in Part 2 of Capital, Volume 3

The capitalists' demand for equal profits on all parts of the capital advanced determines the transformation of value—and by proxy, the living labor expended—into market value in such a way that "capital becomes conscious of itself as social power." (C3, 299)

THESIS: The general rate of profit is a means of social power.

9 Calculation of market prices in Volume 4 of Capital

. . . notes for unfinished sections 5.2 ff. follow in Appendix

10 Appendix: Notes for market value and market prices

Sources: Capital Vol. III and *Theories of Surplus Value* (MECW)

For a fixed choice of period and for each $i = 1, \ldots, N$, let $\mathcal{C}_1, \ldots \mathcal{C}_N$ be pairwise disjoint sets of commodities respectively produced by N **branches** (or **spheres of production**).

Let

c_i = average **constant capital invested in** $\omega \in \mathcal{C}_i$, $\qquad$ v_i = average **variable capital embodied in** $\omega \in \mathcal{C}_i$,

s_i = average **surplus-value embodied in** $\omega \in \mathcal{C}_i$, $\qquad$ $p_i = s_i$ = average **mass of profit embodied in** $\omega \in \mathcal{C}_i$.

Let $s_i' = s_i/v_i$ be the (**average**) **rate of surplus-value** and let $p_i' = s_i/(c_i + v_i)$ be the (**average**) **rate of profit**

for branch i.

For a real-valued random variable $x : \Omega \to \mathbb{R}$ defined on a set Ω, the **expected value** (or **mean**) of x is

$$\mathbb{E}(x) = \sum_{\omega \in \Omega} x(\omega) \cdot \mathcal{P}\big(x = x(\omega)\big).$$

Regard p' as a random variable. Let $\mu_{p'} = \mathbb{E}(p')$ be the **general rate of profit**. Then

$$\mu_{p'} = \mathbb{E}(p') = \sum_i p'_i \cdot \mathcal{P}(p' = p'_i) = \sum_i p'_i \frac{c_i + v_i}{\sum_j c_j + v_j}.$$

Let $(\text{CP})_i = \tau_{c_i} c_i + v_i$ be the **cost price for branch** i, where $\tau_{c_i} = \frac{\text{used up } c_i}{c_i}$ is the **rate of consumption of constant capital in branch** i.

Let $(\text{PP})_i = \big(\tau_{c_i} + \mu_{p'}\big)c_i + \big(1 + \mu_{p'}\big)v_i$ be the **price of production for branch** i.

We make the simplifying assumption that $\tau_{c_i} = 1$ for all i; so

$$(\text{CP})_i = c_i + v_i, \qquad\qquad (\text{PP})_i = \big(1 + \mu_{p'}\big)(c_i + v_i).$$

Let $(\text{MP})_i(\omega)$ be the **individual market price** and $(\text{MV})_i(\omega)$ be the **individual market value** of a commodity ω produced by branch i. Regard these functions as random variables $\mathcal{C}_i \to [0, \infty)$.

Let $\mu_{(\text{MV})_i}$ denote the (**average**) **market value in branch** i,

$$\mu_{(\text{MV})_i} = \mathbb{E}\big((\text{MV})_i\big) = \sum_{\omega \in \mathcal{C}_i} (\text{MV})_i(\omega) \cdot \mathcal{P}\big((\text{MV})_i = (\text{MV})_i(\omega)\big).$$

10.1 Analytic propositions

Proposition (Tendency of the rate of profit to fall). If s' is bounded above and $\dfrac{c}{v} \to \infty$, then $p' \to 0$.

Proof. As $\dfrac{c}{v} \to \infty$, $p' = \dfrac{s}{c + v} = \dfrac{s'}{\frac{c}{v} + 1} \to 0$ if s' is bounded above. $\qquad\square$

Proposition (General rate of profit). $\mu_{p'} = \dfrac{\sum s_i}{\sum c_j + v_j}.$

Proof. $\mu_{p'} = \sum_{p'=p'_i} p'_i \cdot \mathcal{P}(p' = p'_i) = \sum_i \dfrac{s_i}{c_i + v_i} \cdot \dfrac{c_i + v_i}{\sum_j c_j + v_j} = \dfrac{\sum s_i}{\sum c_j + v_j}.$ $\qquad\square$

Proposition. $(\text{PP})_i$ is a function of $c_1, \ldots, c_N, v_1, \ldots, v_N, s_1, \ldots, s_N$.

Proof. Define

$$g(c_1, \ldots, c_N, v_1, \ldots, v_N, s_1, \ldots, s_N) = \frac{\sum s_i}{\sum c_j + v_j} = \mu_{p'}.$$

Define

$$h(c_i, v_i, s_i) = (1 + \mu_{p'})(c_i + v_i)$$

for $i = 1, \ldots, N$. Then

$$(\text{PP})_i = h\big(c_i, v_i, g(c_1, \ldots, c_N, v_1, \ldots, v_N, s_1, \ldots, s_N)\big)$$

for $i = 1, \ldots, N$. $\qquad\square$

Proposition. If branch i has organic decomposition $\dfrac{\mu_c}{\mu_v}$ and $\tau_c = 1$, then $(\text{PP})_i - (\text{CP})_i = s_i$.

Proof. Suppose branch i has organic decomposition μ_c/μ_v. Then

$$\frac{s_i}{c_i + v_i} = p_i' = \mu_{p'},$$

so

$$(\text{PP})_i - (\text{CP})_i = (\text{CP})_i \, \mu_{p'} = (c_i + v_i)\frac{s_i}{c_i + v_i} = s_i.$$

$\qquad\square$

Proposition. $\sum(\text{PP})_i = \sum c_i + v_i + s_i.$

Proof.

$$\sum(\text{PP})_i - (\text{CP})_i = \mu_{p'} \sum(c_i + v_i) = \sum_j p_j' \frac{c_j + v_j}{\sum_i c_i + v_i} \sum(c_i + v_i) = \left(\frac{\sum s_j}{\sum c_i + v_i}\right) \sum_i (c_i + v_i) = \sum s_i.$$

$\qquad\square$

Proposition. $\mu_{(\text{MV})_i} = c_i + v_i + s_i$ for each branch i.

Proof. For each i, $\mu_{(\text{MV})_i}$ is the average market value of a commodity produced in sphere i. But under the assumption that commodities sell at their values, $c_i + v_i + s_i$ is the average market value in sphere i:

$$(\text{MV})_i = c_i + v_i + s_i.$$

$\qquad\square$

10.2 Economic (synthetic) propositions

Proposition (Malthus's Rule). Equal capitals yield equal profits.

Proposition. The general rate of profit changes only over the long term.

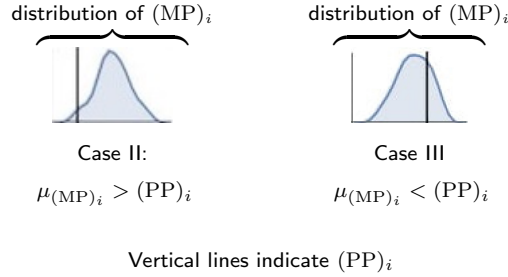**Proposition.** The rate of surplus-value tends to equalize across all branches.

Proposition. Competition distributes the total social capital between branches in such a way that

$$\sum (\text{PP})_i = \left(1 + \mu_{p'}\right)(c_i + v_i)$$

for all i.

Proposition. Writing $\diamondsuit$ for "is empirically correlated with," the following three statements hold.

CASE I:	$\mu_{(\text{MP})_i} = (\text{PP})_i$	$\diamondsuit$	conditions of production under average conditions predominate	$\diamondsuit$	demand $=$ supply
CASE II:	$\mu_{(\text{MP})_i} > (\text{PP})_i$	$\diamondsuit$	conditions of production under worse conditions predominate	$\diamondsuit$	supply $<$ demand
CASE II:	$\mu_{(\text{MP})_i} < (\text{PP})_i$	$\diamondsuit$	conditions of production under better conditions predominate	$\diamondsuit$	supply $>$ demand

distribution of $(\text{MP})_i$ distribution of $(\text{MP})_i$

Case II:
$\mu_{(\text{MP})_i} > (\text{PP})_i$

Case III
$\mu_{(\text{MP})_i} < (\text{PP})_i$

Vertical lines indicate $(\text{PP})_i$

Proposition (Marx contra Say). Overproduction is always possible.

Proposition (Equalization of the General Rate of Profit). Due to competition, which causes investment to circulate between branches,

$$\left|\mu_{(\text{MP})_i} - (\text{PP})_i\right| \to 0$$

in each branch i over time.

Proposition. Organic decomposition is explanatory of market prices and the rate of profit is not.